Numerical modelling and simulation of floating oil storage tanks considering the sloshing effect

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Preface

This report is performed at Department of Marine Technology at Norwegian University of Technology and Science (NTNU), which would also serve as part of the master thesis at Chalmers University of Technology (CTH).

This project is cooperate with SINTEF Ocean (formerly MARINTEK) in connection with their project with NUS on multi-purpose floating structures. The main goal of the project is to develop a time-domain model for motion response analysis of a single floating oil storage tank, and a few interconnected tanks under wave loads considering the effect of sloshing of the internal flow based on the potential flow theory. The main task of this thesis is to develop the time-domain retardation functions of the radiation forces using the frequency-domain hydrodynamic results for both external and internal flow, and use them for time-domain simulations in the software SIMO, developed by SINTEF Ocean. A comparison in terms of platform motions and loads in moorings and connectors will be made for the cases with and without the consideration of internal flow.

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Xiao Liu
Abstract

Floating oil storage facilities are proposed to store oil products with combined modular floating oil storage tanks (FOST) on near-shore area, which can save land space for the land-limited countries. When designing these floating tanks, there are some difficulties in performing hydrodynamic analysis. One of these difficulties comes from the internal fluid, i.e. when there exists internal fluid, sloshing may have significant influence on the motions of the floating tanks, however, the effects of sloshing is only considered to be significant on the structure loads and its effects on the global motion are ignored, which can introduce uncertainties when the motions are of a concern.

In this project, the coupling effects of FOST motion and internal liquid sloshing of FOST are studied by both frequency-domain and time-domain approaches. Two cases are set to reveal the effects of sloshing on the motions of FOST, one is filled cases equipped with internal fluid, and the other is solid case provided with equivalent solid mass in the internal tank for comparison.

In the frequency-domain analysis, the hydrodynamic results including added-mass coefficients, potential-damping coefficients, and transfer functions, of solid case and filled case are obtained by the established 3D panel model. In the time-domain analysis, for the filled case, the tank-motion simulation and internal-liquid-sloshing-motion simulation are performed independently. The calculation of external-liquid motion was regarded as same as that for the solid case. For the internal liquid sloshing, the retardation function was derived based on added mass of inner liquid and the hydrostatic stiffness correction due to inner free surface. The corresponding force and moments were calculated by MATLAB programming, and regarded as external excitation in the body motion equation. During the global motion analysis of the filled case, the sloshing motion is coupled with the tank motion, and the effects of internal liquid sloshing are evaluated. The time-domain analysis were performed in regular and irregular incident wave conditions.

The internal liquid sloshing has a significant effect on the surge and pitch of the tank motion, relatively, the influence on heave motion is weak in some frequency regions. For the surge and pitch motion, especially for the pitch motion, the peak frequency can be shifted because of the
sloshing effect. Also, when the encounter frequency at the resonance range, the phase difference between the sloshing motion and the wave excitation leading to the dramatic reduction of the global motion. The retardation function was computed according to the added mass of internal liquid as well as the hydrostatic stiffness correction due to inner free surface. By comparing the RAO obtained and that obtained from the frequency-domain hydrodynamic analysis, the present method by regarding the internal-liquid sloshing as the external excitation to the body motion is proven to be reliable. The global motion of the hydrodynamic response on regular wave conditions is mild, and the maximum motion is less than 3m, which illustrates the design is reasonable. Even though the simulation on DNV-SIMO only based on the potential damping has some limitations, the direct simulation carried out by SIMO still is able to consider the effect on sloshing in some extent. There is a little problem on the recalculated added mass, and for this point, further work needs to be studied.

The results from this thesis can not only provide important input to the design of FOST in the next stage but also provide the meaningful reference to similar sloshing-motion coupling problem.
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Chapter 1

Introduction

1.1 Background

During the different processes of petroleum production, crude oil/processed oil needs to be stored in storage tanks. Some companies would store crude oil, and sell them when the oil prices increase. Meanwhile, some countries would store oil for strategic reserve. Also, in some regions, there is extra oil which more than their needs and the oil will be stored in the storage tanks.

The storage tanks can have different types and sizes, once the crude oil is drilled, it needs to be refined and processed into what we use in our daily life, like petrol, diesel or paraffin. During this process, oil storage tanks are inevitable before the oil to be moved to other places for further refinement.

However, the onshore oil storage tanks are usually space-consuming. To save land space, especially for countries with very limited land area and with long coastlines, i.e. Singapore, Japan, offshore floating storage tanks are proposed and considered as a good way to store oil, oil products and/or liquefied natural gas. Theses tanks can not only help to increase more storing spaces utilizing near-shore water area, but also reduce the explosive risk from human factors as these facilities will be built far away from the land. To provide some preliminary information, this project work will focus on the hydrodynamic response analysis for a near-shore stationary
floating hydrocarbon storage tank.

In conventional analysis for offshore structures with internal fluid, the effects of inner free surfaces are considered for stability analysis and hydrostatic analysis, while the dynamic effects of the sloshing inside the liquid container is normally ignored. Usually, much of the hydrodynamic research on sloshing focuses on estimating the peak values and duration of impact pressure for the structural analysis. For the storage tank, its inner storage compartment will be partially filled during its whole operating period including offloading time under the sea wave conditions. The large inner free surface and sloshing effect can not be neglected, in other words, the wave-induced responses of the storage tank are significantly influenced by the motion of the internal liquid (e.g. oil, liquefied natural gas), and consequently, the liquid inside is in turn altered by the ship motions. This process is of great concerns for the Floating Oil Storage Tank (FOST).

1.2 Approach

The internal sloshing in the FOST is excited by the body motion, and the induced sloshing will also disturb the global motion of tank in return. Therefore, the coupling effect is inevitable, which means it is necessary to investigate both problems of sloshing and tank motion simultaneously.

In this project, to investigate the coupling effect of sloshing and body motion, a combined frequency-domain and time-domain numerical model have been built based on potential flow theory. To reveal the influence of sloshing, a comparison has been performed for the filled cases with internal fluid and solid case with equivalent solid mass in the internal tank.

First of all, in the frequency-domain, the tank diffraction and radiation problem is solved from a three-dimensional panel method, and all the hydrodynamic coefficients of tank motions are obtained. Considering the inverse Fourier transformation of the frequency-domain results, a time-domain model is established in DNV-SIMO project team (2015). It is noted that SIMO calculates the retardation function depending on the potential damping. However, there is no
Therefore, to perform the time-domain analysis of filled case, it is necessary to calculate the retardation function of internal liquid. The corresponding sloshing force and moments are computed based on convolution integral, which are then imported into the SIMO and taken as an external excitation to the body motion. As a return, the obtained body motion from time-domain analysis is taken as the motion of internal liquid motion to iterate for the further
simulation. Under regular wave conditions, the motion response amplitude operators (RAOs) can be evaluated. And this can be compared with that obtained in frequency domain. Also, the time-domain analysis is performed with irregular wave conditions considering the sloshing effect.

The main methodology can be briefly summarized as the flowchart in figure 1.1.

1.3 Literature survey

To investigate the coupling effect of internal liquid sloshing and tank motion, there are lots of brilliant professors and students have been working on this field. With the assumption of linear sloshing flow, some studies perform the simulation in frequency domain. To include the mooring force of floating tank, time-domain motion analysis are also computed in some reports. To perform the analysis under irregular wave condition, Rognebakke and Faltinsen (2003) considered the convolution formulation, and discussed the treatment details of the retardation function for the external problem. Newman (2005) analyzed the coupled motions by extending the panel code in WAMIT. The finite-difference method is introduced by Kim (2002) to simulated the sloshing flow; and Kim et al. (2007) adopted the impulse-response-function (IRF) method to investigate the ship motion. LI et al. (2012) carried out the numerical computation by Open Field Operation and Manipulation (OpenFOAM) and managed to consider the coupling effect between wave, sloshing, and ship motion. To assessed the coupled sloshing impact pressure, Park et al. (2010) performed the numerical analysis in time-domain, and taking the results as a input data for the simulation. Except for the numerical studies, some experiments are also carried out to investigate the effect of inner sloshing. Lee et al. (2007) observed the influence of internal liquid filling level on coupling effects, and Zhao et al. (2013) found the response due to sloshing effect is sensitive to the wave excitation frequency.
1.4 Structure of the report

The present paper is organized as follows. The potential theories and methods of convolution integral for hydrodynamic analysis in both frequency domain and time domain are introduced in Chapter 2. The implementation of the panel model and the hydrostatic results obtained in frequency domain are described in Chapter 3. In Chapter 4, the time-domain analysis is performed, and the internal liquid sloshing retardation function as well as the corresponding forces and moments are performed, also the response simulated under regular and irregular wave conditions are given and discussed. Some conclusions and the future work are summarized in Chapter 5.
Chapter 2

Theory

In this project, the analysis is performed in frequency domain and time domain, both of them are based on the potential flow theory. The details of the potential theory are based on the book of *Sea Loads on Ships and Offshore Structures* (Faltinsen, 1993) and the corresponding lecture notes *Sea Loads* (Greco, 2012), as well as the WAMIT User Manual (WAMIT, 2013). The part of convolution integral is according to the DNV-SIMO Theory Manual (project team, 2015), and lecture notes of stochastic methods by Zhen Gao (Gao, 2016).

2.1 Definition of the coordinate systems

In the analysis of the body motion in waves, two coordinate systems are introduced (Marin, 2017a) (Gate, 2017): earth fixed global system $o_g - x_g y_g z_g$ as well as the body fixed system $o_b - x_b y_b z_b$, as shown in 2.1.

The origin of the earth fixed global system is at the waterline, and the axis $o_g z_g$ is positive in vertically upward direction, and the signs of rotation are right handed.

To determine the location of a body, it is necessary to describe its position and its orientation in global coordinate system. There are six degrees of freedom (6-DOF), and the wave-induced motions of the body are introduced: surge, sway, heave, roll, pitch, and yaw, as shown in figure
2.2 (Marin, 2017b). The first three motions are used to describe the translatory displacements along x, y and z directions, and the last three motions are the rotations of the structure.

The body fixed coordinate system is fixed with respect to the floating body, and will move with body when its position and orientation are changing. Its origin is normally defined on the body's center of gravity (COG) or on the project of COG on the free surface.
2.2 Potential flow theory

2.2.1 Basic equations

In potential flow, the fluid is assumed inviscid and incompressible, and the fluid motion is irrotational. Then velocity potential is introduced, and it has to satisfy the Laplace equation:

$$\nabla^2 \phi = 0 \quad \text{in} \quad \Omega$$  \hspace{1cm} (2.1)

To find the solution of the velocity potential of irrotational, incompressible fluid motion, the solution of the Laplace equation with connected boundary conditions have to be solved. The corresponding boundary conditions mainly focus on:

- Kinematic boundary condition
- Dynamic free-surface condition

Due to the impermeability condition, the kinematic boundary condition for a fixed body in moving fluid is:

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on} \quad S_{SB}$$  \hspace{1cm} (2.2)

The equation 2.2 is normally used for the sea bottom as no fluid enters or leaves the body surface.

If the body is moving with velocity $\mathbf{U}$, the boundary equation is shown as:

$$\frac{\partial \phi}{\partial n} = \mathbf{U} \cdot \mathbf{n} \quad \text{on} \quad S_B$$  \hspace{1cm} (2.3)

where, $\mathbf{U}$ can be any type of the body velocity, which means for the various point on the body surface, $\mathbf{U}$ shall be different.

Assume there is no forward speed for the structure, the current is zero, and wave amplitude is small comparing to the body dimension, under such conditions the velocity potential is proportional to the wave amplitude, which is named as a linear theory.
When fluid particles on free surface, \( z - \zeta(x,y,t) = 0 \), and remain at here \( D(z - \zeta)/Dt = 0 \), the free-surface kinematic condition is:

\[
\frac{\partial \phi}{\partial z} = \frac{\partial \zeta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \zeta}{\partial y} \quad \text{on} \quad z = \zeta(x,y) \tag{2.4}
\]

The free-surface dynamic boundary condition is valid when the pressure is equal to the ambient pressure \( p_a \):

\[
g\zeta + \frac{\partial \phi}{\partial t} + \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 \right] = 0 \quad \text{on} \quad z = \zeta(x,y) \tag{2.5}
\]

The combined free surface condition is:

\[
\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = 0 \tag{2.6}
\]

### 2.2.2 Motions equations

According to the Newton's second law, the motion of the body is:

\[
\sum_{k=1}^{6} m_{kj} \ddot{\eta}_j = F_k \quad k = 1...6 \tag{2.7}
\]

With the assumption of the inviscid fluid, the forces and moments are obtained by integrating the pressure over the wetted surface of the body:

\[
P = -\rho \frac{\partial \phi}{\partial t} - \rho g z \tag{2.8}
\]

Due to linearized potential flow theory, the superposition principle is assumed and the potential \( \phi \) can be decomposed in terms of the fundamental physical effects involved in the fluid interaction. The linear wave body interaction problem could be split into two sub-problems: diffraction problem and radiation problem (Faltinsen, 1993).

In the diffraction problem, the body is fixed and interacts with the incident waves, the velocity potential is divided as the potential for the incident waves \( \phi_0 \) and the diffraction potential
The wave excitation loads equal to the Froude-Kriloff loads plus the diffraction loads, where Froude-Kriloff loads is due to the water enters the body with normal velocity \( \partial \phi_0 / \partial n \) and cause hydrodynamic loads on the body, while the diffraction loads is due to the body’s presence causing a flow which is associated with \( \phi_D \) to recover the body impermeability. The total excitation forces are obtained by integrating the incident wave dynamic pressure and the diffraction dynamic pressure over the mean wetted hull surface:

\[
F_{exc,k}(t) = -\int_{SOB} \rho \frac{\partial \phi_0}{\partial t} n_j dS - \int_{SOB} \rho \frac{\partial \phi_D}{\partial t} n_j dS \quad k = 1...6
\]

In the radiation problem, the body is forced to oscillate in the 6 degree of freedoms with frequency, the moving body will generate radiated waves, and is subjected to pressure forces. The dynamic forces can be identified as added mass, added damping and restoring terms. The added mass and added damping are connected to the dynamic forces caused by the body motions while the restoring force is generated by static pressure due to the buoyancy varying. The total radiation forces are also obtained by integrating the pressure on the mean wet surface. Therefore, the total potential velocity can be obtained by:

\[
\phi(x, y, z, t) = \phi_0(x, y, z, t) + \phi_D(x, y, z, t) + \phi_R(x, y, z, t)
\]

In frequency domain, based on the linearized potential flow theory, the steady-states sinusoidal motions for the rigid body can be written as:

\[
\sum_{k=1}^{6} [(m_{jk} + A_{jk}) \dot{\eta}_k + B_{jk} \dot{\eta}_k + C_{jk} \eta_k] = F_j e^{-i\omega t} \quad j = 1...6
\]

where, \( m_{jk} \) are the generalized mass matrix components for the structure, \( A_{jk} \) is the hydrodynamic added mass matrix, \( B_{jk} \) is the hydrodynamic damping matrix, \( C_{jk} \) is the hydrostatic restoring matrix, \( \eta \) is the body motions and \( F_j \) are the external force vector induced by the waves.
Reconsidered the new mass $M$ to be as follow:

$$M = m_{jk} + A_{jk}$$  \hspace{1cm} (2.12)

and the natural frequency is:

$$\omega = \sqrt{\frac{C_{jk}}{M}} = \sqrt{\frac{C_{jk}}{m_{jk} + A_{jk}}}$$  \hspace{1cm} (2.13)

The equation 2.11 can also be written as matrix form:

$$[m + A(\omega)]\ddot{\eta} + B(\omega)\dot{\eta} + C\eta = F$$  \hspace{1cm} (2.14)

### 2.3 Frequency domain analysis

#### 2.3.1 Body motions in regular waves

In a linear system at steady state condition, the floating body in the calm water is stable, which means the weight of structure equals to that of displaced water in the mean position. And the hydrodynamic loads is only with respect to the wave-body interactions.

To neglect the time dependence, the frequency-dependent excitation forces with an amplitudes $\zeta$ are shown as the complex form:

$$F(t) = \mathbb{R}\{\zeta X(\omega, \beta)e^{-i\omega t}\}$$  \hspace{1cm} (2.15)

The response is written as:

$$\eta(t) = \mathbb{R}\{\eta_a(\omega, \beta)e^{-i\omega t}\}$$  \hspace{1cm} (2.16)

The equation of body motion for the linear wave 2.7 becomes:

$$\sum_{j=1}^{6} [-\omega^2(M_{kj} + A_{kj}(\omega)) + i\omega B_{kj}(\omega) + C_{kj}]\eta_{ja} = \zeta_a X(\omega, \beta)$$  \hspace{1cm} (2.17)
To evaluate the motion, the excitation and radiation loads are required, the element of the mass matrix including the body mass, the moments of inertia, the products of inertia, and the coordinate of the center of mass. The mass matrix $\mathbf{M}$ is displayed as the inertial force (the first three rows) as well as the inertial moments (the last three rows).

The elementary inertial moment is shown as:

$$\mathbf{dM} = \mathbf{r} \times \mathbf{dF} = \mathbf{d} \mathbf{m} (\mathbf{r} \times \ddot{\mathbf{s}})$$

(2.18)

where, $\mathbf{r}$ is a vector from the objective point to the COG and $\mathbf{dF}$ is the inertial force vector related to the acceleration in each direction.

The motion can be estimated by the transfer function, which specifies the motion amplitude per unit incident-wave amplitude as well as the phase of the motions relative to the incident waves:

$$\mathbf{H}(\omega, \beta) = \eta_a / \xi_a = [-\omega^2 (\mathbf{M} + \mathbf{A} (\omega)) + i \omega \mathbf{B}(\omega) + \mathbf{C}]^{-1} \mathbf{X}(\omega, \beta)$$

(2.19)

$\mathbf{H}(\omega, \beta)$ is a complex number in which the angle denotes the phase relationship between the excitation and the ship motions, while the amplitude $|\mathbf{H}(\omega, \beta)|$ is known as the response amplitude operator (RAO), which provides the response amplitude per unit wave amplitude.

### 2.3.2 Body motions in random sea state

With linear theory, the irregular waves can be regarded as a sum of regular waves in sine- and/or cosine functions with random phases.

Considering an irregular sea state by a certain spectrum $S(\omega)$, whose two parameters i.e. peak period $T$ and significant wave height $H_{1/3}$ are constant, corresponding wave elevation can be expressed as:

$$\zeta = \sum_{j=1}^{N} A_j \sin(\omega_j t + k_j x + \epsilon_j)$$

(2.20)
where,
\[ \frac{1}{2} A^2_j = S(\omega_j) \Delta \omega \]  \hspace{1cm} (2.21)

in which the interval frequency \( \Delta \omega = (\omega_{\text{max}} - \omega_{\text{min}})/N \).

Each single regular-wave component has the amplitude \( A_j = \sqrt{2S(\omega_j)\delta \omega} \), frequency \( \omega_j \), and random phase \( \epsilon_j \). Therefore, by linearly superposing the steady-state response of single wave components, the response is:

\[ \sum_{j=1}^{N} A_j |H(\omega_j)| \sin((\omega_j t - k_j t + \delta(\omega_j) + \epsilon_j) \right) \]  \hspace{1cm} (2.22)

where \( \delta(\omega_j) \) is the phase angle associated with the response of the corresponding incident regular wave.

In the limit when \( N \to 0 \), and \( \Delta \omega \to \infty \), the variance of response can be estimated as:

\[ \sigma_r^2 = \int_0^\infty S(\omega)|H(\omega)|^2 d\omega \]  \hspace{1cm} (2.23)

### 2.4 Time domain analysis

#### 2.4.1 Motion equations

A time-domain model of a floating oil storage tank was established in DNV-SIMO based on the frequency-domain hydrodynamic results obtained in WAMIT.

For sinusoidal motion, the equation of motion can be written as:

\[ \mathbf{M} \ddot{\mathbf{x}} + \mathbf{B} \dot{\mathbf{x}} + \mathbf{D}_1 \dot{\mathbf{x}} + \mathbf{D}_2 f(\dot{\mathbf{x}}) + \mathbf{C} \mathbf{x} = \mathbf{q}(t, \mathbf{x}, \dot{\mathbf{x}}) \]  \hspace{1cm} (2.24)

Here, \( \mathbf{D}_1, \mathbf{D}_2 \) are the linear damping matrix, and quadratic damping matrix, \( \mathbf{q}(t, \mathbf{x}, \dot{\mathbf{x}}) \) is the excitation force vector. In this project, the viscous damping is treated as the quadratic damping.
CHAPTER 2. THEORY

To find the solution of the motion equation, the method of convolution integral can be used.

\[ m + A(\omega)\ddot{x} + B(\omega)\dot{x} + C(\omega)x = f'(t) = q - D_2f(\dot{x}) - D_1x \]  

(2.25)

Considering the hydrodynamic coefficients obtained in frequency domain, the equation becomes:

\[ A(\omega)\ddot{x} + B(\omega)x = f(t) = f'(t) - Cx - m\ddot{x} \]  

(2.26)

Suppose the force on the right hand varies sinusoidally at individual frequency, and the equation of dynamic equilibrium in frequency domain is,

\[ \left[-\omega^2(A(\omega) + i\omega B(\omega))\right]X(\omega) = F(\omega) \]  

(2.27)

or

\[ [i\omega A(\omega) + B(\omega)]i\omega X(\omega) = F(\omega) \]  

(2.28)

\(A(\omega)\) and \(B(\omega)\) are frequency-dependent added mass and damping, and the relations are:

\[ A(\omega) = A_\infty + a(\omega) \]  

(2.29)

\[ B(\omega) = B_\infty + b(\omega) = b(\omega) \]  

(2.30)

where, \(A_\infty = A(\omega = \infty)\), and \(B_\infty = B(\omega = \infty) = 0\), so the equations of motion can be rewrite as:

\[ -\omega^2(m + A_\infty)X(\omega) + [i\omega a(\omega) + b(\omega)]i\omega X(\omega) + CX(\omega) = F^{exc}(\omega) \]  

(2.31)

Introducing the Fourier and inverse Fourier transform,

\[ F.T. \rightarrow X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} x(t)e^{-i\omega t} \, dt = \Im[x(t)] \]  

(2.32)

\[ I.F.T. \rightarrow x(t) = \int_{-\infty}^{+\infty} X(\omega)e^{i\omega t} \, d\omega = \Im^{-1}[X(\omega)] \]  

(2.33)
CHAPTER 2. THEORY

With the Inverse Fourier transform, the equation is shown as:

\[(m + A_\infty)\ddot{\mathbf{x}}(t) + \int_{-\infty}^{+\infty} [i\omega a(\omega) + b(\omega)] i\omega X(\omega) e^{i\omega t} d\omega + C\mathbf{x}(t) = f_{\text{exc}}(t) \tag{2.34}\]

2.4.2 Retardation function

The second term in the motion 2.34 is known as the Fourier (or Inverse Fourier) transform of multiplication, and it is regarded as a convolution of Fourier transforms.

Considering the Parseval’s theorem,

\[2\pi \int_{-\infty}^{+\infty} F(\omega) \cdot \overline{G(\omega)} d\omega = \int_{-\infty}^{+\infty} f(\tau) \cdot \overline{g(\tau)} d\tau \tag{2.35}\]

Supposing

\[F(\omega) = (i\omega a_{ij}(\omega) + b_{ij}(\omega)) e^{i\omega t} \tag{2.36}\]
\[\overline{G(\omega)} = i\omega X_j(\omega) \tag{2.37}\]

Therefore, the second term can be shown as:

\[\int_{-\infty}^{+\infty} [i\omega a_{ij}(\omega) + b_{ij}(\omega)] e^{i\omega t} \cdot i\omega X_j(\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\tau) \overline{g(\tau)} d\tau \tag{2.38}\]

So

\[f(\tau) = \mathcal{F}^{-1}[F(\omega)] = \int_{-\infty}^{+\infty} (i\omega a_{ij}(\omega) + b_{ij}(\omega)) e^{i\omega t} e^{i\omega \tau} d\omega \tag{2.39}\]
\[= \int_{-\infty}^{+\infty} (i\omega a_{ij}(\omega) + b_{ij}(\omega)) e^{i\omega (t+\tau)} d\omega \tag{2.40}\]

Define the retardation function \(k(\tau)\), which can be computed by a transform of the frequency-
dependent added-mass and damping coefficients:

$$k(\phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (i\omega a(\omega) + b(\omega)) e^{i\omega \tau} d\omega$$  \hspace{1cm} (2.41)

therefore,

$$f(\tau) = 2\pi k_{ij}(t + \tau)$$  \hspace{1cm} (2.42)

and the second term is:

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} 2\pi k_{ij}(t + \tau) \dot{x}_j(-\tau) d\tau$$  \hspace{1cm} (2.43)

So the motion equation can be written as:

$$(m + A_0)\ddot{x}(t) + \int_{-\infty}^{+\infty} k(t - \tau) \dot{x}(\tau) d\tau + Cx(t) = F^{exc}(t)$$  \hspace{1cm} (2.44)

With the fact that $a(-\omega) = a(\omega)$ and $b(-\omega) = b(\omega)$, retardation function can be shown as:

$$k(\phi) = \frac{1}{\pi} \int_{0}^{+\infty} (b(\omega) \cos \omega \tau - \omega a(\omega) \sin \omega \tau) d\omega$$  \hspace{1cm} (2.45)

**Solid case**

For a floating tank under the condition of hydro-statically stable, when experiencing small oscillations due to external forces or moments, the motion at that time only depends on the forces or moments happened before that time. This is known as the causality of radiation effect, $k(\tau) = 0$ for $\tau < 0$, which means there is no memory effect of the future before the steady state.

For $\tau' = -\tau$ with $\tau > 0$, $k(\tau') = 0$.

$$\frac{1}{\pi} \int_{0}^{+\infty} (b(\omega) \cos \omega(-\tau) - \omega a(\omega) \sin \omega(-\tau)) d\omega = 0$$  \hspace{1cm} (2.46)
It is easy to know that, for $\tau > 0$:

$$\frac{1}{\pi} \int_0^{+\infty} b(\omega) \cos \omega \tau d\omega = -\frac{1}{\pi} \int_0^{+\infty} \omega a(\omega) \sin \omega \tau d\omega$$  \hspace{1cm} (2.47)

This means, for $\tau' = \tau$, with $\tau > 0$, the retardation function can be computed either by frequency-dependent added mass or potential damping coefficients as follows:

$$k(\tau) = \frac{2}{\pi} \int_0^{+\infty} b(\omega) \cos \omega \tau d\omega = -\frac{2}{\pi} \int_0^{+\infty} \omega a(\omega) \sin \omega \tau d\omega$$  \hspace{1cm} (2.48)

Also, the frequency-dependent added mass and damping can be found by the retardation function, and their relations are called as Kramers-Kronig relations:

$$a(\omega) = -\frac{1}{\omega} \int_0^{+\infty} k(\tau) \sin \omega \tau d\tau$$  \hspace{1cm} (2.49)

$$b(\omega) = \int_0^{+\infty} k(\tau) \cos \omega \tau d\tau$$  \hspace{1cm} (2.50)

Filled case

The above condition refers to the solid case or the external flow, which means there is no internal fluid, so the sloshing effect does not work. However, for the internal flow, as there is no potential damping, it could not get to stable state, which means the causality of radiation is not applicable for the internal flow.

Therefore,

$$B(\omega) = 0$$  \hspace{1cm} (2.51)

$$k(-\tau) = k(\tau)$$  \hspace{1cm} (2.52)

The retardation of internal flow can be expressed as:

$$k(\tau) = \frac{1}{\pi} \int_0^{+\infty} (-\omega a(\omega) \sin \omega \tau) d\omega$$  \hspace{1cm} (2.53)
The Kramers-Kronig relations is not valid at here, and the added mass can be computed as:

\[ a(\omega) = -\frac{2}{\omega} \int_0^{+\infty} k(\tau) \sin \omega \tau \, d\tau \]  

(2.54)

### 2.4.3 Internal fluid sloshing effects

When internal liquid exists with free surface in one floating body, sloshing may be stimulated under specific motion modes of that body. When the wave periods or frequencies equals or close to the natural frequency of the limited region, it will cause large fluid motions because of the limited damping related with the fluid motion, and this resonance phenomenon is characterized as sloshing. Significant sloshing plays a role in slamming, wave breaking or other violent motions associated with high local pressures and large global loads.

With the sloshing effect, the diffraction damping of the internal tank does not exist because of the absence of incoming wave \((Hu \ et \ al., 2016)\). And as the internal free surface will also move with tank motion, the boundary conditions of free-surface dynamic (equation 2.5) for the internal tank will be changed as:

\[ g\xi + \frac{\partial \phi}{\partial t} = \dot{Z}_0 \]  

(2.55)

In equation 2.55, \(\dot{Z}_0\) is due to the velocity of the vertical motion (heave, roll, pitch) of the overall ship. As \(\dot{Z}_0\) is not equal to zero, the non-homogeneous solution will be obtained. Therefore, the motion equation will be presented as follow:

\[ \sum_{k=1}^{6} \left[ (M_{jk} + A_{jk} + A_{jk-tank})\ddot{\eta}_k + (B_{jk} + B^*)\dot{\eta}_k + (C_{jk} - C_{jk-tank})\eta_k \right] = F_j e^{-i\omega t} \]  

(2.56)

In the equation, \(A_{jk-tank}\) represents the added mass due to the internal liquid motion. The linear equivalent damping coefficient is is often considered of the viscous effect, shown as the term of \(B^*\). And \(C_{ij-tank}\) is the hydrostatic restoring matrix only for tanks and it is usually negative, therefore the overall restoring force will be decreased correspondingly.

To solve the problems of the internal sloshing effects, the resulting forces and moments are
considered as the exterior domain outside of the tank. The velocity potential in the tank is:

\[
\phi = i\omega \sum_{j=1}^{6} \xi_j \phi_j, \tag{2.57}
\]

where \(\xi_j\) is the amplitude of motion.

To find the solution of the velocity potential \(\phi\) in the tank, the computation is performed simultaneously with the potential in the external fluid domain, and an extended linear system which contains both the exterior and interior fluid domains is being used. However, suppose the assumption that the separate fluid domains are independent and there is no influence between them, then the elements, which the row and column refer to different domains, on the extended influence matrix are zero.

However, attention should be paid on vertical motions.

The hydrostatic restoring coefficients of the tank are given by:

\[
C_T(3,3) = \rho g \int \int_{S_T} n_3 dS \tag{2.58}
\]
\[
C_T(3,4) = \rho g \int \int_{S_T} yn_3 dS \tag{2.59}
\]
\[
C_T(3,5) = -\rho g \int \int_{S_T} xn_3 dS \tag{2.60}
\]

The volume \(\forall\) can be calculated by integrating over the mean wetted surface of the tank \(S_T\):

\[
\forall = \int \int_{S_T} n_1 x dS = \int \int_{S_T} n_2 y dS = \int \int_{S_b} n_3 (z - Z_T) dS \tag{2.61}
\]

In WAMIT (WAMIT, 2013), the hydrostatic coefficients of the body and tank are computed separately, so when calculating volume and stiffness coefficients, the tank panels is neglected, and their values are same with or without a tank.

Supposing the tanks are related to the body, there are extra term added for the tank, the
CHAPTER 2.  THEORY

Hydrostatic restoring coefficients are shown as:

\[ C(3, 3) = C(3, 3) + \left( \rho_T / \rho \right) \int \int_{S_T} n_3 dS \] (2.62)

\[ C(3, 4) = C(3, 4) + \left( \rho_T / \rho \right) \int \int_{S_T} y n_3 dS \] (2.63)

\[ C(3, 5) = C(3, 5) - \left( \rho_T / \rho \right) \int \int_{S_T} x n_3 dS \] (2.64)

For the solid case, with no internal fluid, the added mass and damping coefficients are evaluated by integrating the pressure over the external hull surface.

As for the filled case, due to the existence of internal fluid, to get the added mass coefficients, the pressure has to be integrated over both the external hull surface and internal tank surfaces respectively. As there is no radiation damping from the tanks, the damping coefficients should be zero for the internal tank, which means the damping coefficients are the same between the solid case and filled case. For the vertical motions, the fictitious hydrostatic stiffness has an influence on the added mass.

For the heave motion, the boundary conditions can be shown as:

\[ \frac{\partial \phi}{\partial n} = n_3 \quad \text{on} \quad S_B \] (2.65)

\[ k \phi - \frac{\partial \phi}{\partial z} = 0 \quad \text{on the free surface} \] (2.66)

Therefore the heave potential is obtained as:

\[ \phi_3 = z + 1/k \] (2.67)

where \( z \) is the local vertical coordinate over the tank free surface, and \( 1/k \) is a constant which is required by the free-surface condition. \( k \) is wave number and can be determined by dispersion function \( \omega^2 = gk \).
CHAPTER 2. THEORY

The heave added mass coefficient is:

\[ A_{33} = \rho \int \int_{S_T} n_3 \phi_3 dS = \rho \mathcal{V}_T + C_T(3,3)/(g k) \] (2.68)

\[ A_{34} = \rho \int \int_{S_T} (yn_3 - zn_2) \phi_3 dS = \rho \mathcal{V}_T y_c + C_T(3,4)/(g k) \] (2.69)

\[ A_{35} = \rho \int \int_{S_T} (zn_1 - xn_3) \phi_3 dS = -\rho \mathcal{V}_T x_c + C_T(3,5)/(g k) \] (2.70)

For the floating tank with volume \( \mathcal{V}_T \) and waterplane area \( S_T \), the added mass is \( \rho \mathcal{V}_T + S_T/k \), and contribution from the term \( S_T/k \) can be canceled by the hydrostatic restoring stiffness.

2.4.4 Approach for time-domain analysis

Solid case

For the solid case, the motion of hull is:

\[ (m + a^\infty) \ddot{\mathbf{x}} + \int_{-\infty}^{+\infty} k(t - \tau) \dot{\mathbf{x}}(\tau) d\tau + C \mathbf{x} = F^{\text{exc}}(t) \] (2.71)

Filled case

The FOSL with partially filled liquid, leads to the effect of sloshing. The internal liquid will be excited by the motion of hull, and also the hull's motion will be affected by the sloshing-induced excitation as well as the external wave-induced excitation. Therefore, the two aspects should be considered simultaneously.

And the tank motion is presented as:

\[ M \ddot{\mathbf{x}} + B \dot{\mathbf{x}} + C \mathbf{x} = F^{\text{external}} + F^{\text{internal}} \] (2.72)

The external force contains the diffraction and radiation forces, and the internal force is mainly due to the effect of sloshing.
Take the convolution integral into consideration, the motion is rewritten as:

\[(m + A^\infty)\ddot{x}(t) + \int_{-\infty}^{+\infty} k(t - \tau)\dot{x}(\tau) d\tau + Cx(t) = F^{exc}(t) + F^{slosh}(t)\]  \hspace{1cm} (2.73)

Here, the \(F^{exc}(t)\) refers to the wave-excitation force acting on the external hull, and \(F^{slosh}(t)\) is the sloshing-induced force acting on the internal hull.

The motion of internal liquid sloshing is:

\[(A_{L}^\infty)\ddot{x}(t) + \int_{-\infty}^{+\infty} k_{L}(t - \tau)\dot{x}(\tau) d\tau + Cx(t) = F^{slosh}(t)\]  \hspace{1cm} (2.74)

Here, the terms with subscript \(L\) refer to the properties of the internal liquid.

And the motion of the total tank in the filled condition becomes:

\[(m + a^\infty + a_{L}^\infty)\ddot{x}(t) + \int_{-\infty}^{+\infty} k(t - \tau)\dot{x}(\tau) d\tau + \int_{-\infty}^{+\infty} k_{L}(t - \tau)\dot{x}(\tau) d\tau + (C + C_{L})x(t) = F^{exc}(t)\]  \hspace{1cm} (2.75)

During the simulation, the convolution of internal liquid is computed in the Matlab programming, and then exported as the external force into the DNV-SIMO. The time-domain analysis is then carried out to compute the total motion of filled case.
Chapter 3

Frequency-domain analysis

In the simulation of the coupling effect of the FOST’s body motion and tank sloshing, it requires the frequency-domain results prior to conversion to time-domain. In frequency domain, the diffraction/radiation analysis is performed based on the established three-dimensional panel model, after which the hydrodynamic coefficients, i.e. added mass coefficients and potential-damping coefficients, as well as the transfer function. The linear/drift wave forces are obtained during the analysis.

3.1 Properties of floating storage oil tank (FOST)

The FOST is a concrete tank, with dimensions 35 m × 35 m × 20 m. It consists of an external rectangular tank and an internal cylindrical tank, and the principle dimensions of FOST are listed in table 3.1.

As the FOST is designed to store various oil products in the near-shore area around Singapore, where water is shallow and the water depth is assumed as 30 m. And the sea water density is set as 1025 kg/m³, the oil density is 870 kg/m³. The geometry of the floating oil storage tank is presented in figure 3.1.
Table 3.1: Detailed tank dimension

<table>
<thead>
<tr>
<th></th>
<th>unit</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank external side length</td>
<td>m</td>
<td>35</td>
</tr>
<tr>
<td>Tank internal diameter</td>
<td>m</td>
<td>32</td>
</tr>
<tr>
<td>Total tank height</td>
<td>m</td>
<td>20</td>
</tr>
<tr>
<td>Tank bottom thickness</td>
<td>m</td>
<td>1.5</td>
</tr>
<tr>
<td>Height of tank roof</td>
<td>m</td>
<td>5.3</td>
</tr>
<tr>
<td>Internal tank height</td>
<td>m</td>
<td>13.2</td>
</tr>
<tr>
<td>Internal tank wall thickness</td>
<td>m</td>
<td>0.7</td>
</tr>
<tr>
<td>External tank wall thickness</td>
<td>m</td>
<td>0.7</td>
</tr>
<tr>
<td>Total weight of tank roof</td>
<td>m</td>
<td>2.35E+06</td>
</tr>
<tr>
<td>Total tank weight (empty)</td>
<td>m</td>
<td>9.65E+06</td>
</tr>
<tr>
<td>Vertical center of gravity of tank (empty)</td>
<td>m</td>
<td>7.82</td>
</tr>
</tbody>
</table>

Figure 3.1: Floating oil storage tank

The coordinate system has an origin at the still water level and the coordinate system has been defined, as shown in fig 3.2. The rigid body motions in linear sea-keeping consist of three translations along x-, y-, z- axis as surge, sway and heave, and three rotations about x-, y- z-axis as roll, pitch and yaw respectively.
3.2 Frequency-domain model establishment

To perform frequency-domain analysis, it is inevitable to build the frequency-domain model.

With the given geometry dimension, a panel model is established in SESAM-GeniE, and a structural model discretized into finite elements is also built to simulate the mass distribution of the tank. Both models are imported into SESAM-HydroD for the operation of hydrostatic and hydrodynamic analysis. Details regarding modeling using SESAM-HydroD and GeniE can be found in the user menus (Norske, 2013a) and (Norske, 2013b).

The establishment of the model in SESAM-GeniE is based on the following process:

- Specify the properties i.e. material, cross section and plate thickness
- Create geometry model
- Build panel model
- Establish the full model
- Make a compartment
It is noted that the FOST has an internal tank, which is closed with a top plate, and filled with oil. So this closed internal tank is automatically defined as a compartment in GeniE.

With the structure model, creating an analysis activity with the mesh size as 1m, and the model by GeniE is shown as fig 3.3.

Since the panel model and the structural model have been generated in SESAM-GeniE, importing them into the SESAM-HydroD. Before the Genie-made models imported to HydroD, a location and a hydro model are created, where water density is 1025 $kg/m^3$, kinematic viscosity is 1.19E-06 $m^2/s$, the oil density in the internal tank is 870 $kg/m^3$, and water depth is set as 30m as the water filed near Singapore. The panel model characterized the wetted surface, which will be used to calculate buoyancy of the tank. Attention should be paid that both panel model and structural model are translated the depth of draft in the negative Z direction, as the reference in HydroD is the free surface, but in GeniE is the baseline.

The model generated in HydroD is shown in fig 3.4.
3.2.1 Loading conditions

Supposing the FOSL is equipped with partially filled liquid, leading to the change of drafts. Also, with the existing of internal fluid, there is free surface, which will not only further reduce the stability; but also cause sloshing effect, which plays a role on the extra added mass and damping force, influencing the tank hydrodynamic.

To investigate the inner fluid effect, comparison has been performed for filled cases which with 40% internal fluid and solid case which equipped with the equivalent solid mass in the internal tank.

The details of loading conditions can be seen in table 3.2. VCG is the center of gravity of tank, it is with reference to the calm water level, positive upward and negative downward.

<table>
<thead>
<tr>
<th>Case</th>
<th>Empty tank</th>
<th>40% filled</th>
<th>40% solid</th>
<th>Full tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draft (m)</td>
<td>7.7</td>
<td>10.64</td>
<td>10.64</td>
<td>15.06</td>
</tr>
<tr>
<td>Damping b44&amp;b55 (N*s/m)</td>
<td>2.4E+08</td>
<td>2.9E+08</td>
<td>2.9e+08</td>
<td>3.9E+08</td>
</tr>
<tr>
<td>VCG (m)</td>
<td>0.13</td>
<td>-3.83</td>
<td>-3.83</td>
<td>-7.23</td>
</tr>
</tbody>
</table>
CHAPTER 3. FREQUENCY-DOMAIN ANALYSIS

3.3 Hydrodynamic results

3.3.1 Response amplitude operators (RAOs)

The responses of FOST with the partially filled oil and equivalent solid weights are compared.

Figure 3.5 shows the RAO of global surge motion in the solid case and filled case respectively. Sloshing effect on surge motion can be distinguished obviously. The RAO in surge motion for the filled case is a little smaller than that of the solid case at the excitation frequencies. It is obvious to find, for the filled case, there is a point that the surge response is almost zero, and this is due to the excitation frequency equals to the first mode of sloshing frequency.

However, at the part where the excitation frequency is much larger than the first mode of sloshing frequency, it can be seen that the RAO of filled case is larger than that of the solid case.

According to (Kim et al., 2007), near the resonance frequency, the wave-excitation and sloshing-induced moments has about 180° phase difference, which leads to a significant decrease on the global motions. However, the excitation frequency move away from the first mode sloshing frequencies (resonance frequency), and the phase difference tends to 0°, which indicates the

![Figure 3.5: RAO of surge motion with wave heading of 0°](image)
sloshing effect have an influence on the global motions.

In figure 3.6, the heave RAO under the condition of filled case and solid case are presented. However, there is no difference can be found between them, which illustrates the sloshing effect has no significant influence on heave motion. But, it is noted that when the frequency closes to zero, the heave RAO tends to be 1, which shows that under the small frequencies, the tank is hydrostatically following the incident wave free surface for excited waves.

Figure 3.6: RAO of heave motion with the direction of 0°

The pitch RAO illustrates significant effects of the internal fluid sloshing on the tank global motion, the single peak for solid case is split into two separated smaller peaks for filled case, which is shown in figure 3.7. This phenomenon is relevant to the phase between the internal sloshing and the incident wave elevations, and correspondingly, the peak amplitude RAO in filled case is much decreased. Very likely, at the cancellation point, the frequency which is smaller than the sloshing resonance frequency, the external wave loads cancel with the internal wave loads.
3.3.2 Linear/drift wave forces

The linear/drift wave force is shown in the figure 3.8, and their behavior are almost same for the solid case and the filled case, that only one resonant peak can be identified for the both cases. This is due to the fact that the wave force is calculated based on the shape of the hull, and the
generated wave energy are almost same outside of the body. The two curves are overlapping at the low-frequency and high-frequency fields. However, the amplitude difference at the middle part may cause by the effect of internal liquid sloshing.

### 3.3.3 Added-mass coefficients

With HydroD, the added mass are treated as dependent only on frequency. Because of the FOST is symmetry about xz- and yz- plane, many terms of added mass are zero or close to zero especially for terms which are not on the diagonal. Moreover, due to axial-symmetry, the added mass of surge and sway motion are the same, and that of roll and pitch motion are also the same. In figure 3.9, there is six principles added mass matrix, the difference of added mass under the sloshing effect can be detected.

Sloshing effect has a significant influence on the surge, sway, roll, and pitch. For these motions, their resonant frequency is at about 8.3 \( \text{rad/s} \). At resonance, the added mass coefficients rise rapidly and tend to infinity, which corresponds significant fluctuation of RAO. But resonance peaks are significantly over-predicted because viscous effects are not included. At the resonant frequency, where added mass is approaching to \(+\infty\) while the RAO close to the zero. With the frequency increasing a little more, the added mass drops and goes to negative infinity, which is due to the reason that the negative added mass cancels the body mass. (Newman, 2005)

### 3.3.4 Potential-damping coefficients

Due to axial-symmetry of the geometry, which has been said before, the damping of surge equals to that of sway, and roll damping is the same with the pitch damping. In figure 3.10, there is the damping matrix of the surge, heave, roll and yaw.

The potential damping goes to zero at the frequency is zero or is \(+\infty\), which means the body can not generate any waves. It is known that to propagate waves, the free surface must be both a horizontal and a vertical velocity component. However, at \( \omega \to 0 \) or \( \omega \to \infty \), there can not be both a horizontal and a vertical velocity at the free surface.
Figure 3.9: Frequency-dependent added mass coefficients
Figure 3.10: Frequency-dependent potential damping coefficients
Chapter 4

Time-domain simulation

In chapter 3, the three-dimensional (3D) diffraction/radiation panel program based on potential theory has been established in the frequency domain, and the response amplitude operator, hydrodynamic coefficients and the drift forces of filled case and solid case have been obtained. According to the results under each specific frequency, tank sloshing coupling with body motions can be simulated in the time domain. For the filled case, the simulation of internal liquid sloshing effect is performed by using convolution integral, then the calculated internal liquid sloshing force and moment are taken as the external excitation to the body motion. Then, using the software DNV-SIMO, time-domain analysis of regular and irregular wave analysis are carried out, and the sloshing effect is investigated by comparing the results of the filled case and of the solid case.

4.1 Time-domain model introduction

4.1.1 Time-domain model establishment

In the frequency-domain analysis, all the hydrodynamic coefficients, as well as the corresponding forces, have been estimated. After which, by exporting them into the DNV-SIMO, the time-domain model is shown in 4.1.
In the model, there are four springs tied in the tank, and they are modeled by the fixed force elongations. The stiffness of the spring is initially set as $1 \times 10^5 N/m$, and the pretension is 1000N.

Figure 4.1: The time-domain model established in SIMO

### 4.1.2 Time-domain models for filled case

For filled case, to simulate the body motions considering the sloshing effect in time domain, there are two different models are introduced as shown in table 4.1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>1. Calculate the retardation function of internal fluid;</td>
<td>Regular wave condition</td>
</tr>
<tr>
<td></td>
<td>2. Obtain the corresponding forces and moments;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Import them as the external excitation to the body motion;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Perform the time-domain analysis in DNV-SIMO</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>1. Do direct simulation based on DNV-SIMO programming;</td>
<td>Irregular wave condition</td>
</tr>
<tr>
<td></td>
<td>2. Revise the hydrostatic stiffness on heave motion;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Fix the added mass at infinity frequency;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Perform the time-domain analysis in DNV-SIMO</td>
<td></td>
</tr>
</tbody>
</table>

The main difference between each other is that the model 1 calculated the retardation function of inner sloshing by convolution integral in Matlab programming, and taken the corresponding
forces and moments as external excitation to the global motion. However, in model 2, the retardation function is calculated by DNV-SIMO itself, by revising the hydrostatic stiffness and added mass in infinity frequency to perform the further time-domain analysis.

### 4.2 Retardation function computation

To study the coupling effect of sloshing and body motion, both the solid case and filled case are taken into consideration. However, for the filled case of model 1, the corresponding retardation function and convolution force are calculated by MATLAB programming. And for the solid case and filled case of model 2, the retardation functions are calculated by SIMO automatically.

#### 4.2.1 Truncation error

The frequency-dependent added mass and potential damping coefficients are calculated for each specific frequency. The frequency varies from $0.2027 \text{ rad/s}$ to $6.2832 \text{ rad/s}$, and the frequency interval is not a constant. The distribution of added-mass and potential damping coefficients obtained in frequency-domain are more intensive near the peak values, and less intensive at the high-frequency (low-period) field, which can be seen as the discrete points in figure 4.2 or in table 4.2.

<table>
<thead>
<tr>
<th>Periods (s)</th>
<th>1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39</td>
</tr>
<tr>
<td>6.1</td>
<td>6.2 6.3 6.4 6.5 6.6 6.7 6.8 6.9 7 7.05 7.1 7.15 7.2 7.25 7.3 7.35 7.4 7.45 7.5 7.55 7.6 7.65 7.7 7.75 7.8 7.85 7.9 7.95 8 8.1 8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 9 9.1 9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9 10</td>
</tr>
<tr>
<td>10.5</td>
<td>11 12 12.5 13 13.5 14 15 16 17 18 19 20 21 22 23 24 25</td>
</tr>
<tr>
<td>27</td>
<td>29 31</td>
</tr>
</tbody>
</table>

Table 4.2: Periods-set of frequency-domain analysis

The retardation function is calculated based on the equation 2.45. The added mass and the damping curves have to be calculated at constant frequency intervals, and the results are more
accurate with small intervals. Especially for the filled case, the interval needs to be pretty small to recognize the high spike which generated by internal liquid sloshing.

Therefore, the method of interpolation is introduced. The added mass and damping curves are interpolated based on the original period/frequency set as given in table 4.2, the applied frequency integral is 0.001 rad/s.

Meanwhile, in the actual computation of retardation function, the integral is generally carried out in the finite frequency range, (from 0.2027 rad/s to 6.2832 rad/s), a truncation error is inevitable.

\[
Error(\tau) = \frac{1}{\pi} \int_0^{0.2027} (b(\omega)\cos\omega \tau - \omega a(\omega)\sin\omega \tau)d\omega + \frac{1}{\pi} \int_{6.2832}^{+\infty} (b(\omega)\cos\omega \tau - \omega a(\omega)\sin\omega \tau)d\omega
\]

(4.1)

According to (Kim et al., 2007), the highest frequency has a significant influence on retardation function. It is found that a wider range of frequency improves the accuracy of results, even the number of frequency is not so critical if the highest-frequency is large enough. Therefore, to obtain an accurate solution, it is necessary to extend the frequency range, and this can be achieved by considering the variation trend of added mass and damping toward high frequency and zero frequency.

For example, at low-frequency, added mass can remain its value at the known-minimum-frequency until zero-frequency. As for damping, a linear function from zero could be used to describe the tend, due to the fact that potential damping is zero at \( \omega = 0 \).

At high frequencies, the added mass and damping tend to their asymptotic values at a rate of \( \omega^{-2} \), Pérez and Fossen (2008). The potential damping tends to be zero at infinite frequency. It is difficult to predict the added mass at infinity frequency accurately, and the time domain solution is sensitive to the consequences of the infinite frequency added mass. In this project, considering the maximum frequency of 6.2832 rad/s (which is big enough), the frequency is not to be extended further. Figure 4.2 illustrates the interpolation of the solid case, the red lines show the frequency-dependent added mass and potential damping interpolated based on the
Figure 4.2: Interpolation of frequency-dependent added-mass and damping for solid case
results from the frequency-domain hydrodynamic results, as well as the extrapolation at low-frequency.

4.2.2 Retardation function calculation

In chapter 2, it is known that the retardation function can be computed by either a potential damping coefficient or an added mass for the solid case or external liquid. In the DNV-SIMO program (project team, 2015), the retardation function is calculated by frequency-dependent damping coefficient, and this method is due to the causality of the radiation effect. However, in this project, considering the floating oil tank would be filled with internal fluid, and there is no damping inside the tank, which means it is impossible to get a steady state. Therefore, the causality effects could not apply for the internal liquid of the filled case. To compute the retardation function of the inner liquid in the filled case, a Matlab program is developed.

Solid case

For the solid case, the retardation function is computed respectively by using damping coefficients, added mass, as well as the combination of damping and added mass coefficients. As the internal fluid is not considered, it is expected that there is no significant difference between the three different computations, as well as the computation obtained in SIMO.

Figure 4.3 shows the retardation function computed in three different ways and that obtained in SIMO (in SIMO, retardation function is calculated based on the potential damping coefficients). As expected, the curves of retardation functions are almost the same except at the starting point. According to the three different method computations, as given in equation 2.45 and equation 2.48, at \( t = 0 \), the expression for the damping term leads to \( \cos(\omega t) = 1 \), while the added mass term is \( \sin(\omega t) = 0 \), which explains why the retardation function calculated by added mass at \( t = 0 \) trends to 0, that the retardation function by damping trends to 1. While, if applying both the potential damping and added mass, the retardation function at \( t = 0 \) is the mean value of that computed only by damping or added mass coefficients.
Figure 4.3: Retardation function computed in three different ways and that obtained by SIMO for solid case.
Figure 4.4: The added mass and damping coefficient recalculated by retardation function for solid case
Considering Kramers-Kronig relations, equation 2.49 and 2.50, the added mass and potential damping coefficients can be reevaluated by the calculated retardation function. The added mass and damping are calculated by different retardation functions and comparing to the initial added mass and damping. The results are shown in figure 4.4, where the black dashed line is the initial data. It can be found that these curves are almost overlapping, their difference is due to the different retardation function at the starting point, leading to the different integration area, which has a further effect on the final results.

**Filled case**

For the filled case, the computation is divided as the internal part and external part. And the external part is just the same as the solid case, so the attention is focused on the internal part. The damping coefficient is zero for the internal tank, as there is no radiation from the tank. This means in the equation of the retardation function 2.45, the damping term is zero, only the term of added mass is considered, and is shown as:

\[
k(t)^{\text{internal}} = \frac{1}{\pi} \int_{0}^{+\infty} (-\omega a(\omega)^{\text{internal}} \sin \omega t) d\omega
\]  

(4.2)

The added mass is divided as the internal added mass and the external added mass, both of them includes the added mass at infinite frequency.

\[
A^{\text{total}} = A^{\text{internal}} + A^{\text{external}}
\]

\[
= a^{\text{internal}} + a^{\text{internal}}_{\infty} + a^{\text{external}} + a^{\text{external}}_{\infty}
\]

(4.3)

(4.4)

However, it is noted for the vertical motion, the frictions hydrostatic stiffness have contribution to the added mass, the internal added mass is shown as:

\[
a^{\text{internal}}_{33} = A^{\text{total}}_{33} - a^{\text{external}}_{33} - a^{\text{external}}_{33,\infty} - a^{\text{internal}}_{33,\infty} - C_{33} / g \]

(4.5)

Inserting the corresponding added mass into equation 4.2, and the retardation function for
Figure 4.5: Retardation function computed by added mass for filled case
Figure 4.6: Added mass of internal fluid recomputed by retardation function of filled case
different motions can be obtained. Due to the symmetry of the tank, some terms of the retardation function are zero and only retardation function in surge, heave and pitch are plotted as in figure 4.5. The computation time is set as 1200 seconds, and the retardation function tends to be steady at the end of simulation time.

The similarity to the solid case, the added mass can also be recomputed by the calculated retardation function, the added mass of internal fluid are presented in figure 4.6.

4.2.3 Sensitively study

To detect the influence of frequency interval on the simulation, take the surge motion as an example, the sensitively study is performed for the solid and filled case.

Solid case

For the simulation of solid case, it is expected that the frequency interval has no significant influence on the results, thus a big-difference comparison interval is set as 0.1 rad/s.

Figure 4.7: Retardation function calculated by different frequency integrals for solid case

Figure 4.7 shows that the retardation function is not so sensitive to the frequency interval as
expected. The corresponding added mass and potential damping coefficients (in figure 4.8) which are recalculated by the retardation function are also not influenced, and they are almost same with that obtained by $\Delta \omega = 0.001 \text{rad}/s$.

![Figure 4.8: Added mass coefficients recalculated by different frequency integrals for solid case](image)

**Filled case**

For the filled case, the simulation is predicted to be sensitive to the frequency interval. Due to the internal liquid sloshing, a high spike is induced in the added mass curve. This means the interval should be small enough to identify the spike, then the retardation function can be convergence to continue further work.

To verify this assumption, the frequency interval $0.005 \text{rad}/s$ is chosen to do the calculation. It appears that for $\Delta \omega = 0.005 \text{rad}/s$, the retardation function does not converge well as shown in figure 4.9, comparing to the results generated by $\Delta \omega = 0.001 \text{rad}/s$ shown in figure 4.5.

The interval is set as $0.05 \text{rad}/s$, and the period is about 1200s, which means the cycle of retardation function calculation is 1200 seconds, and as shown in the equation 2.54. The plotting is a symmetrical distribution, however, the retardation function can not converge well within 600s, and influencing the further added mass recalculation as seen in figure 4.10.
Figure 4.9: Retardation function calculated with frequency interval = 0.005 rad/s for filled case

Figure 4.10: Added mass recalculated by frequency integral=0.05 rad/s for filled case
4.3 Decay test

The added horizontal stiffness \( k(N/m) \) is modeled as \( 1.0e + 05N/m \), so the mean drift distance can be expected by the stiffness and mean drift force:

\[
x = \frac{F}{K}.
\]  

(4.6)

According to the small angle assumption, the angle between the spring and the decay distance should be smaller than 5 degrees, so the spring distance can be expected.

The natural period is obtained according to:

\[
T_n = 2\pi \sqrt{\frac{m + M_0}{k}},
\]  

(4.7)

it is noted that the spring stiffness \( k \) is for the whole system, instead of the single spring stiffness.

To identify the natural period of each mode of motion, decay tests are performed in SIMO by giving exciting forces at the initial stage. The time history of the motion can also be estimated based on the response curve, and then both natural period and damping ratio can be calculated by:

\[
\delta = In \frac{A_1}{A_2},
\]  

(4.8)

\[
n = \frac{\delta}{T_a},
\]  

(4.9)

\[
\omega_n = \sqrt{n^2 + \frac{2\pi}{T_a}},
\]  

(4.10)

and

\[
\xi = \frac{n}{\omega_n}.
\]  

(4.11)
Figure 4.11: The external force and the corresponding mean drift distance of decay test
A viscous/quadratic damping is necessary to take into consideration especially for pitch and heave in this case, and in practice, it is expressed as the ratio of critical damping which can be calculated by:

\[ C_r = 2m\omega_N, \tag{4.12} \]

here, the natural frequency is \( \omega_N = \sqrt{\frac{k}{m+M_o}} \) according to the 4.7.

Therefore, the external force of decay test and its corresponding mean drift distance in the motion of surge, heave, and pitch are shown in figure 4.11.

<table>
<thead>
<tr>
<th>Motion</th>
<th>Natural period (s)</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>47.16</td>
<td>2.96%</td>
</tr>
<tr>
<td>Heave</td>
<td>6.75</td>
<td>8.12%</td>
</tr>
<tr>
<td>Pitch</td>
<td>9.75</td>
<td>8.18%</td>
</tr>
</tbody>
</table>

Table 4.3: Identified natural periods and damping ratio for surge, heave and pitch

From the decay test results in SIMO, the natural period and damping ratio can be easily calculated. The identified natural periods of surge, heave and pitch are 47.16s, 6.75s and 9.75s respectively which are consistent with the corresponding natural periods obtained from solid-case RAOs that can be found in figure 4.12. And the damping ratio of these three motions are 2.96%, 8.12% and 8.18%.

### 4.4 Time-domain analysis for regular wave conditions

Since the time-domain model has been established, it is necessary to compare the motion RAOs obtained in time-domain with those obtained from the frequency-domain hydrodynamic analysis in HydroD.

#### 4.4.1 Regular wave conditions

In the frequency-domain analysis, the wave period varies from 0.1s to 31 s, but the time interval is not constant, and the points are more intensive around the natural period.
In the time-domain simulation, the regular wave is decided first. The wave direction is 0°, the phase is 0s, and wave amplitude is 1m. The regular wave set specified with 18 various wave periods, which is shown in table 4.4.

<table>
<thead>
<tr>
<th>Case number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (s)</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>7.5</td>
<td>8</td>
<td>8.5</td>
<td>9</td>
</tr>
<tr>
<td>Frequency (rad/s)</td>
<td>2.09</td>
<td>1.57</td>
<td>1.26</td>
<td>1.05</td>
<td>0.90</td>
<td>0.84</td>
<td>0.79</td>
<td>0.74</td>
<td>0.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case number</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (s)</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Frequency (rad/s)</td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
<td>0.42</td>
<td>0.35</td>
<td>0.30</td>
<td>0.26</td>
<td>0.23</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 4.4: Regular wave conditions

4.4.2 RAO comparison for frequency domain and time domain

Solid case

The time-domain analysis under regular wave conditions is performed, and the motion RAOs are compared with that obtained from the frequency-domain analysis. The comparison of amplitude and phase of RAOs of surge, heave, and pitch are shown in the figure 4.12.

From figures 4.12, very approximate results can be found between the RAOs obtained from time domain and frequency domain, which shows that both methods are applicable for the solid cases. However, considering the time cost of time-domain analysis, the frequency-domain analysis may be the better choice for the RAOs’ prediction if only linear responses are concerned.

Filled case

For the filled cases, the retardation function of internal liquid sloshing has been calculated, and the calculated sloshing force and moment are taken as the external excitation to the body motion. The overall trends of RAOs from frequency domain and time domain analysis shows good agreement in figure 4.13. Some differences of amplitude can be found at the natural period of each motion which is mainly due to the differences in the input damping ratio.
Figure 4.12: RAOS obtained in frequency-domain and time-domain for solid case
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Figure 4.13: RAO obtained in frequency domain and time domain for filled case
In frequency domain, the potential damping and the corresponding added mass coefficients are obtained by potential theory. In order to get more accurate results in the frequency domain, taking viscous damping into consideration would be helpful. Meanwhile, in time domain, as there is no potential damping inside the tank, with the effect of internal liquid sloshing, only added mass is involved in the simulation.

However, under the natural period of sloshing, the value seems to be quite consistent, especially in pitch RAO, the knock out point can be distinguished obviously. These consistent results prove that the adopted time domain model is applicable for solving the problem of importing sloshing effect in SIMO analysis. Based on this model, further study under irregular wave conditions can also be performed in SIMO to predict the extreme responses considering the effects of sloshing.

4.4.3 Tank sloshing coupling with ship motions

The RAOs of filled case are compared with that of the solid case from time domain analysis in SIMO as shown in 4.14. The results are similar as that of the former chapters, the sloshing effects are significant in pitch RAOs and can be found in surge RAOs. As what has been expected, there is no sloshing effects in heave motions. This can be more clear in time histories of surge, heave and pitch motions.

Results under the selected wave periods of 7 s are shown as 4.15. From these figures, obvious differences can be found between solid cases and filled cases both of which are from time-domain analysis in SIMO. At T=7s, the surge and pitch motion are suppressed by sloshing effects, while the heave motions are almost the same in both cases.

4.5 Time-domain analysis for irregular wave conditions

4.5.1 Wave condition

As the real sea states are random, wave spectrum is always used to describe the features of the random sea states. JONSWAP spectrum is one of the most common spectrum used in offshore
engineering. Its function is shown as follow:

Figure 4.15: Time history of motion with regular wave at T=7s
Figure 4.14: RAO of solid case and filled case in time-domain
\[ [H]S_j(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\frac{5}{4} \left(\frac{\omega_p}{\omega}\right)^4\right] \]

\[ r = \exp\left[-\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2}\right] \]

The figure of wave spectrum is shown as 4.16. Considering the water-field near Singapore, the peak period is set as 7s (the peak frequency is 0.9 rad/s correspondingly), significant wave height is 2m, and direction is 0°, corresponding to the 100-year storm in Singapore. Once these parameters have been input into SIMO, it will generate the random 3-hour sea states. For the procedure of generating the random waves based on the spectrum, one can refer to the user manual of SIMO.

![Wave spectrum](image)

Figure 4.16: Spectrum of irregular wave

The generated wave elevation is also presented in figure 4.17.
4.5.2 Results in Solid Case

The three-hour time histories of surge, heave and pitch responses in a 100-year storm are shown in figure 4.18. Statistical results are also given as table 4.5: for the extreme wave conditions, maximum heave is only 0.82m, the maximum pitch is 3 degree, the maximum surge motion is 2.86m, while the mean drift can be found to be 0.23m, which shows that the motions of the tank are very moderate even in the extreme conditions. For the storage tank, the moderate motions will help to decrease the requirements when designing the whole facilities including the supporting system such as fenders or moorings.

It should note that with internal fluid, the motions will be different due to the sloshing effects. However, it should be noted that extreme wave conditions have a peak period of 7s and the JONSWAP spectrum is also a narrow band, which means the wave energy is mainly at this period. The sloshing natural period is around 5s, so the sloshing effect under the extreme sea states can not be as significant as that shown in regular waves.
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Figure 4.18: Time-domain analysis with irregular wave conditions of solid case
CHAPTER 4. TIME-DOMAIN SIMULATION

<table>
<thead>
<tr>
<th>Motion/unit</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (m)</td>
<td>2.86</td>
<td>-2.33</td>
<td>0.23</td>
<td>0.85</td>
</tr>
<tr>
<td>Heave (m)</td>
<td>0.82</td>
<td>-0.84</td>
<td>-0.001</td>
<td>0.25</td>
</tr>
<tr>
<td>Pitch (°)</td>
<td>3.00</td>
<td>-2.79</td>
<td>0.003</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table 4.5: Time-domain analysis with irregular wave conditions of solid case

4.5.3 Results in filled case by SIMO

Due to the time limitations, the time-domain analysis under irregular wave conditions using the proposed method is not performed, instead, the direct simulation considering sloshing effects based on DNV-SIMO programming is performed.

And the RAO of filled case in directly calculated in the time domain by SIMO is compared with that obtained in the frequency domain, the comparison is shown in the figure 4.19. It can be seen that the results given by SIMO seem to show the sloshing effect as well, though no significant difference can be found in statistical results from irregular analysis as shown in figure 4.21 and table 4.6, that the maximum surge motion is 2.84m, the maximum heave motion is 1.09, and maximum pitch motion is 2.54°. However, one point must be clarified is in SIMO analysis, the retardation function is calculated by radiation damping, but it is all known that there is no potential damping inside the tank.

To simply investigate this and verify the correctness of SIMO programming, the retardation found by SIMO is used to recalculate the added mass, and check whether the obtained added mass corresponds to the initial added mass. And the result is compared with added mass from WAMIT as shown in figure 4.20. From these results, it can be seen that in SIMO direct simulation, the added mass has a similar trend as WAMIT results which means the sloshing effects has been considered. However, the peaks of added mass a11 and a55 due to sloshing are much smaller in SIMO results than that of WAMIT results. And the added mass a33 from SIMO is significantly different from that of WAMIT results, this may be explained by that the added mass in WAMIT is modified by the hydrostatic stiffness C33 to consider the internal fluid effects as explained in Chapter 2.4.3. However, to comprehensively understand this problem, further investigation is necessary.
Figure 4.19: RAO obtained in frequency domain and time domain by SIMO for filled case
Figure 4.20: Added mass recalculated of filled case
Figure 4.21: Time-domain analysis with irregular wave conditions of filled case
### Table 4.6: Time-domain analysis with irregular wave conditions of filled case

<table>
<thead>
<tr>
<th>Motion</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (m)</td>
<td>2.84</td>
<td>-2.44</td>
<td>0.19</td>
<td>0.91</td>
</tr>
<tr>
<td>Heave (m)</td>
<td>1.09</td>
<td>-1.07</td>
<td>-0.003</td>
<td>0.33</td>
</tr>
<tr>
<td>Pitch (°)</td>
<td>2.54</td>
<td>-2.51</td>
<td>0.001</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Chapter 5

Conclusion and Recommendations

5.1 Conclusion

In this project, the coupling effects of tank motion and internal liquid sloshing of FOST are investigated by frequency-domain and time-domain simulation programs. To reveal the effects of sloshing effects, a comparison has been performed for filled cases (with internal fluid) and the solid case (with equivalent solid mass in the inner tank).

In frequency-domain analysis, the hydrodynamic results including added-mass coefficients, potential-damping coefficients, and transfer functions, of solid case and filled case are obtained by the established 3D panel model.

In time-domain analysis, for the filled case, the tank-motion simulation and internal-liquid-sloshing-motion simulation are performed independently, and the computation of external-liquid motion is regarded as same as that for the solid case. For the inner sloshing, the retardation function is derived based on added mass of inner liquid and the hydrostatic stiffness correction due to inner free surface. The corresponding force and moments are calculated by Matlab programming, and which are regarded as external excitation to the body motion. During the global motion analysis of filled case, the sloshing motion is coupled with the tank motion, and the effect of internal liquid sloshing is evaluated. The time-domain analysis are performed both for regular and irregular wave conditions.
Through whole procedure, some conclusions are made:

The internal liquid sloshing has a significant effect on the surge and pitch of the tank motion, relatively, the influence on heave motion is weak.

For the surge and pitch motion, especially for the pitch motion, the peak frequency can be shifted because of the sloshing effect. Also, when the encounter frequency at the resonance range, the phase difference between the sloshing motion and the wave excitation leading to the dramatic reduction of the global motion.

The retardation function is computed according to the added mass of internal liquid as well as the hydrostatic stiffness correction due to inner free surface. By comparing the RAO obtained in time domain and that obtained from the frequency-domain hydrodynamic analysis, the present method by regarding the internal-liquid sloshing as the external excitation to the body motion is proven to be reliable.

The global motion of the hydrodynamic response on regular wave conditions is mild, and the maximum motion is less than 3m, which illustrates the design is reasonable.

Even though the simulation on DNV-SIMO only based on the potential damping has some limitations, the direct simulation carried out by SIMO still is able to consider the effect on sloshing in some extent. There is a little problem on the recalculated added mass, and for this point, further work needs to be studied.

5.2 Recommendations

Due to limited time, several problems have not been fully discussed or studied in this project. If time allowed, the time-domain analysis under irregular wave conditions using the proposed method should be performed, and compared with that obtained for the solid case, then the internal liquid sloshing effect could be further investigated.

The direct simulation considering sloshing effects based on DNV-SIMO programming is performed for irregular wave conditions in this project. It is known that there is no potential
damping in the internal liquid, however, SIMO performs analysis according to damping and gives quite satisfactory results. This phenomenon is interesting and confusing, so more time could be spent to study how SIMO consider the sloshing effect.
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