Kalman filtering applied to wave spectrum estimation based on measured vessel responses

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Summary

This thesis focuses on the estimation of waves spectrum based on simulated responses by using Kalman filter. This is useful for development of the onboard real time operator guidance.

The whole progress can be divided into two steps, building simulation environment and estimating wave spectrum. The simulated sea state is described by the irregular wave model, which is generated by random wave frequencies. The vessel responses are generated based on simulated waves and transfer functions (RAOs). Meanwhile, the RAOs data are calculated by using closed-form expression.

In estimation part, the encounter wave spectrum can be calculated by using Kalman filter. One and two vessel simulated responses are applied respectively. The key is this method transferring encounter spectrum to wave spectrum when forward speed is involved. Since it is impossible to do the transformation theoretically, a numerical method is applied to solver the problem. The JONSWAY spectrum is used as the sample to obtain the data for the transfer method.

The estimation results are examined in different conditions as one or two responses, with or without forward speed, different wave directions and different generated spectrum. The results show a good practice of the Kalman filter, which is fast and accurate.
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CHAPTER 1

Introduction

In recent years there has been increasing focus on the application of onboard, real-time operator guidance for ships in terms of decision support systems[9]. The spectrum estimation is an essential part of the system.

The method of spectrum estimation introduced in this thesis is based on the concept of wave buoy analogy, where the vessel is considered as a wave buoy and the measured vessel responses data is given by onboard sensors. Forward speed is one of the key problems need to be solved according to this assumption.

To avoid the transformation between time and frequency domains, Kalman filter is introduced to perform the calculation. As known as linear quadratic estimation, Kalman filter is widely used in time series analysis in fields such as signal processing and econometrics.

And the simulation and analysis are based on the MATLAB.

1.1 Literature review

In order to estimate directional wave spectra using measured ship responses, the wave rider buoy analog is described in e.g. [4] [13] [18].

The wave buoy analogy estimates the waves to which the ship responds can be fond in [11].

Applying the filtering into the wave buoy analogy have been reported in e.g. [12] [14], which are similar to the work in this thesis. Where in [12], forward speed is not included, and it will be discussed more in this thesis.

Except the buoy analogy, wave radar is widely used. Experiences with shipboard wave estimation by wave radar systems have been presented in e.g. [16] [17].

1.2 Problem formulation and objectives

The aim of the thesis is to estimate the wave spectrum. The process can be divided into simulation part and estimation part.

First is to simulate the wave environment and ship motions, which requires the information of wave spectrum and transfer functions.

Second is to obtain the estimated waves spectrum based on the simulated responses from first step. Kalman filter is used as the estimation method. The advan-
tages of Kalman filter is low computational capacity requirement and real time. The transformation process is shown in Figure 1.1.

![Figure 1.1: Main principle of the estimation when it is formulated in time domain](image)

The process of the estimation also can be transferred into frequency domain. The results of estimation will be easier to compare with the generated spectrum. As shown in Figure 1.2.

![Figure 1.2: Main principle of the estimation when it is formulated in frequency domain](image)

### 1.3 Structure of thesis

The outline of this report could be concluded as follows:

- Chapter 2 shows the theory for linear wave theory, the wave spectra and the method of generating waves in simulation.
- Chapter 3 introduces the notation and vessel motions. The closed-form is also introduced for RAOs calculation.
- Chapter 4 explains the theory and calculation of the Kalman filter. The method to estimate the wave spectrum with forward speed is introduced at the end of the chapter.
- Chapter 5 presents an example of container vessel S175. The results are discussed in different conditions: single or double responses, with or without forward speed, different wave directions and different generated spectrum. The discussion focuses on the analysis of the results and error reasons.
Chapter 6 summarises the thesis and give conclusions. Suggestions for further work are given at the end of thesis.
CHAPTER 2

Ocean wave theory

The aim of the thesis is to estimate the wave spectrum by using Kalman filter based on measured vessel responses as mentioned before. Before the estimation, simulation environment has to be built in advance to generate the vessel responses. The simulation environment consists of two parts, the vessel data and the wave data. The wave data is more significant to be considered in this case than the vessel data since the thesis is based on simulation environment instead of full-scale measurement.

For the reason above, wave data is chosen to be studied in order to build the sea state for simulation. Since in real sea state, the vessel responses and waves are random and non-repeatable, the wave data has to be generated through wave spectra, which is based on the linear wave theory.

2.1 Regular and irregular waves

![Regular wave definitions](image)

To build the wave environment, both of basic regular and irregular waves should be introduced. Assume a regular wave as an ideal, two-dimensional, sinusoidal signal function \( \zeta_r \), with basic parameters: the wave height \( H \), the wave length \( \lambda \) and the wave period \( T \). The parameters can be found in Figure 2.1.

Consider the function \( \zeta_r \) of the two variables, position \( x \) and time \( t \):

\[
\zeta_r(t, x) = \zeta_\alpha \cos(kx - \omega t) \quad (2.1)
\]

where, \( \zeta_\alpha \) is the wave amplitude, \( k \) is the wave number and \( \omega \) is the wave frequency. And they are defined respectively according to known parameters as,

\[
\zeta_\alpha = H/2 \quad k = 2\pi/\lambda \quad \omega = 2\pi/T
\]
In engineering application, irregular waves can be assumed as superposition of an infinite number of regular waves, with different wave parameters. Where the frequencies of the waves are from $\omega_1$ to $\omega_2$, the irregular wave could be represented as

$$\zeta_{ir}(t, x) = \sum_{n=1}^{\infty} \zeta_{\alpha,n} \cos(k_n x - \omega_n t + \epsilon_n)$$

(2.2)

where the $\zeta_{\alpha,n}$ is the wave amplitude and the $\epsilon_n$ is the phase angle.

### 2.2 Sea state parameters

In real sea state, the wave in one direction can be described as an irregular wave in Eq. (2.2), which is composed of infinite regular waves. The wave heights and periods are different from each other in these regular waves. So the characteristic parameters only can be derived according to the statistics, which will not be discussed much here.

- **Wave height $H$**
  - Mean wave height $\bar{H}$, the mean value of waves.
  - Significant wave height $H_s$, the mean value of the highest third of waves.

$$H_s = 4.0 \cdot \sqrt{m_0}$$

(2.3)

where the n-th order spectral moments are defined by

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega$$

(2.4)

and $m_0$ is the 0-th order moment.

- **Wave period $T$**
  - Mean period of the peaks $T_p$, the mean value of the time between two successive peaks.
  - Mean zero-crossing wave period $T_z$, the mean value of the time between two successive upward or downward zero crossings.

$$T_z = 2\pi \sqrt{\frac{m_0}{m_2}}$$

(2.5)

- **Wave direction $\beta$**
  
  The wave direction can be represented as in Figure 2.2. Assuming that the vessel sails in the same direction as speed $U$.

  - $\beta = 0$ following sea
  - $\beta = 90^\circ$ beam sea
  - $\beta = 180^\circ$ head sea
2.3 Ocean wave spectrum

In Eq. (2.2), wave amplitude $\zeta_{\alpha,n}$ can be presented as $\zeta_{\alpha,n} = H_n/2$. However it is impossible to get the height of all different waves. The normal way is to generate the wave amplitude by statistics method, which is wave amplitude density spectrum.

The energy in a wave is completely described by amplitude and length of the wave. The wave amplitude density spectrum is used to introduce the wave energy.

And there are several typical spectra as following[2],

- Pierson-Moscowitz (P-M) spectrum
The P-M spectrum model describes a fully developed sea (a sea produced by winds blowing steadily over hundreds of miles for several days) by one parameter, the wind speed. The model is written as

$$S_{PM}(\omega) = \frac{A_S}{\omega^5} \exp\left(\frac{B_S}{\omega^4}\right)$$

where

$$A_S = 0.0081, \quad B_S = 0.74\left(\frac{g}{U_{19.4}}\right)^4$$

and $U_{19.4}$ is the wind speed at 19.4m above sea surface. The model can be changed to the ITTC recommended two-parameter spectrum with the knowledge of significant wave height $H_s$ and mean zero-crossing wave period $T_z$.

- ITTC two-parameter (Bretschneider) spectrum which can be expressed as

$$S_B(\omega) = \frac{A_S}{\omega^5} \exp\left(\frac{B_S}{\omega^4}\right)$$

Figure 2.2: Definition of encounter angle[1]
where

\[ A_S = \frac{H_s^2}{4\pi} \left( \frac{2\pi}{T_z} \right)^4, \quad B_S = \frac{1}{\pi} \left( \frac{2\pi}{T_z} \right)^4 \]

- JONSWAP (Joint North Sea Wave Project) spectrum

During the analysis in Joint North Sea Wave Observation Project, it is found that the wave spectrum is never fully developed. For this reason, the Bretschneider spectrum is modified to consider a limited fetch \([9]\). The expression of the JONSWAP spectrum is shown below

\[ S_J(\omega) = 0.658 \cdot C \cdot S_B(\omega) \tag{2.8} \]

and the factor \( C \) is given by

\[ C = 3.3^J, \quad J = \exp\left[ -\frac{1}{2\gamma^2} \left( \frac{\omega T_0}{2\pi} - 1 \right)^2 \right] \tag{2.9} \]

where

\[ \gamma = 0.07, \quad \omega \leq \frac{2\pi}{T_0} \]
\[ \gamma = 0.09, \quad \omega > \frac{2\pi}{T_0} \]

### 2.4 Simulation of ocean waves

With the knowledge of spectrum and irregular wave, ocean waves can be approximately simulated. Consider a wave spectrum \( S(\omega) \) defined in a range of frequencies \([\omega_1, \omega_2]\). At a given frequency \( \bar{\omega} \), the relation between the wave spectrum \( S(\bar{\omega}) \) and the wave amplitude \( \bar{\zeta}_\alpha \) can be expressed as

\[ \frac{1}{2} \bar{\zeta}_\alpha^2 = S(\bar{\omega}) \Delta \omega \tag{2.10} \]

where the \( \Delta \omega \) is the constant difference between two frequencies. Because it is impossible to generate infinite waves in practice, discretisation has to be introduced. The continuous frequency range \([\omega_1, \omega_2]\) can be separated into \( N - 1 \) equal intervals. The arithmetic progression of \( N \) frequencies will be used to construct the ocean waves

Substitute Eq. (2.10) into Eq. (2.2). With the assumption that the waves are two-dimensional in stationary process and without forward speed, the wave elevation of the long-crested irregular sea can be expressed as sum of \( N \) regular waves

\[ \zeta(t) = \sum_{n=1}^{N} \zeta_n \cos(\omega_n t + \epsilon_n) \tag{2.11} \]

\[ = \sum_{n=1}^{N} \sqrt{2S(\omega_n)\Delta \omega} \cos(\omega_n t + \epsilon_n) \]
If \( N = 1 \), separating the wave elevation into quadrature sinusoidal signals, the function of the wave elevation can be written as

\[
\zeta(t) = \zeta \cos(\omega t + \epsilon) = \zeta \cos(\epsilon) \cos(\tilde{\omega} t) - \zeta \sin(\epsilon) \sin(\tilde{\omega}) = \zeta \cos(\epsilon) \cos(\tilde{\omega} t) + \zeta \cos(\epsilon + \pi/2) \sin(\tilde{\omega})
\]

(2.12)

From Eq. (2.9), new variables could be introduced when \( N = 1 \) as

\[
x_1 = \zeta \cos(\epsilon) \\
x_2 = \zeta \cos(\epsilon + \pi/2)
\]

(2.13)

Substitute Eq. (2.10) into Eq. (2.8), the wave elevation can be written as the function of \( \omega \)

\[
\zeta(t) = \sum_{n=1}^{N} x_{1,n} \cos(\omega_n t) + x_{2,n} \sin(\omega_n t)
\]

(2.14)

where the parameters \( x_{1,n} \) and \( x_{2,n} \) are the functions of the wave amplitude \( A_n \), which can be calculated from wave energy spectrum and the phase angle \( \epsilon_n \). \( \epsilon_n \) are random numbers in uniform distribution.

Thus the expression in Eq. (2.14) can be used as the function to create the wave environment by using the data of the wave frequencies \( \omega_n \) and time \( t \).

2.5 Short crested sea

In real sea condition, the waves are travelling in different directions. It is common to describe the sea in short crested wave system. The short crested wave has a small extent of the direction perpendicular to the direction of propagation. It can be seen in Figure 2.3. The directional wave spectrum is

\[
S_{\zeta}(\omega, \nu) = f(s, \nu, \mu) \cdot S_{\zeta}(\omega)
\]

(2.15)

where the \( f \) is spreading function, which is defined as

\[
f(s, \nu, \mu) = A(s) \cdot \cos^{2s} \left( \frac{\nu - \mu}{2} \right)
\]

(2.16)

where \( \nu \) is the primary wave direction (usually the course of ship), \( \mu \) is the secondary wave direction, which is distributed in the range \(-\pi/2 < (\nu - \mu) < \pi/2\). And \( s \) is the spreading parameter and

\[
A(s) = \frac{2^{2s-1} \Gamma^2(s + 1)}{\pi \Gamma(2s + 1)}
\]

(2.17)
Figure 2.3: Directional wave spectrum [7]
The amplitude of a response to an incident wave of unit amplitude is called transfer function or Response Amplitude Operator (RAO). The transfer function can be expressed as a complex number consisting the amplitude information and phase information, which is the amplitude and phase relation between the wave and the response. Thus, the RAOs are the functions of wave frequencies $\omega$ and wave directions $\beta$.

After generating waves of simulation environment, ship motions can be predicted by using transfer function. There are several methods to calculate the transfer function, like strip theory and commercial softwares. Closed-form method is selected in this project because it is easy and fast to calculate thousands of random wave frequencies in one simulation.

The assessment of seakeeping performance in irregular waves can be done by an analogy between the electronics and communications fields. Although this method was developed in 1953 it is still mostly used for assessment of seakeeping performance [9].

In irregular waves, the ship can be treated as the 'black box', which only deals with signals. When the input signal of 'black box' is waves, it is working as electric filter, giving the signal of ship motions as output. Figure 3.1 shows its mechanism as follows,

![Figure 3.1: The electronic filter analogy](image)
3.1 Ship motions and coordinates

First the centre of gravity of the ship is taken as the origin when its motions are studied. By convention, ship’s motion can be represented by a combination of three linear displacements and three rotations, where

- Linear motions: surge, sway and heave.
- Rotational motions: roll, pitch and yaw.

The six components are shown in Figure 3.2 below, and defined with the right handed axis system.

![Figure 3.2: Axe and ship motion definitions][15]

In this case, heave and pitch motions are mainly studied when simulating the ship motions. The basic model can be considered in two-dimensions, and the heave and pitch motions can be generated by using the transfer function in long crested wave sea state.

3.2 Transfer function

Assume a single regular wave like Eq.(2.1) which is applied on the vessel. Transfer function is used to calculate the ship motions from wave data. It is also assumed that a basic equation of ship motions can be expressed as below, with six degrees of freedom.

\[ x_i = x_{0,i} \sin(\omega + \delta_i) \quad (i = 1, 6) \]  

(3.1)

where \( x_{0,i} \) is the motion amplitude, \( \omega \) is the wave frequency and \( \delta_i \) is the phase.

The transfer function is set as a complex number \( H(\omega) = \text{Re}\{H(\omega)\} + i\text{Im}\{H(\omega)\} \). And \( x_i \) are the modulus of transfer function and \( \delta_i \) can be expressed as the argument of transfer function.
\[ x(\omega) = |H(\omega)| \]
\[ = \sqrt{(Re\{H(\omega)\} + Im\{H(\omega)\}) \cdot (Re\{H(\omega)\} - Im\{H(\omega)\})} \]
\[ = \sqrt{H(\omega) \cdot \overline{H(\omega)}} \tag{3.2} \]
\[
\tan[\delta(\omega)] = \frac{Im[H(\omega)]}{Re[H(\omega)]} \tag{3.3}
\]
in which the transfer function can be expressed as
\[ Re\{H_i(\omega)\} = x_i(\omega) \cos[\delta_i(\omega)] \tag{3.4} \]
\[ Im\{H_i(\omega)\} = x_i(\omega) \sin[\delta_i(\omega)] \tag{3.5} \]

### 3.3 Closed-form expression

To simulate a real sea state, a set of random frequencies need to be generated because of the uncertainty of the real sea. It will be more accurate to calculate the RAOs (Response Amplitude Operators) in these different frequencies instead of using the existing data. Closed-form expression is chosen to be applied for calculating response of heave and pitch motions.

Consider the motions of heave(\(w\)) and pitch(\(\theta\))[5]
\[
2\frac{kT}{\omega^2} \dddot{w} + \frac{D^2}{kB\alpha^3\omega} \dot{w} + w = \zeta F \cos(\omega_e t) \tag{3.6}
\]
\[
2\frac{kT}{\omega^2} \dddot{\theta} + \frac{D^2}{kB\alpha^3\omega} \dot{\theta} + \theta = \zeta G \cos(\omega_e t) \tag{3.7}
\]
where
- \(\zeta\) is the wave amplitude
- \(k\) is the wave number, \(k = \omega^2/g\)
- \(B\) is the breadth of vessel
- \(T\) is the draught of vessel
- \(F, G\) are forcing functions, which will be explained below
- \(\omega\) is the wave frequency
- \(\omega_e\) is the encounter wave frequency
\[
\omega_e = \omega - kU \cos \beta
\]
where \(U\) is the forward speed and \(\beta\) is the wave direction. And \(a\) is
\[
\alpha = 1 - Fn\sqrt{kL} \cos \beta \tag{3.8}
\]
where \(Fn\) is Froude number, \(Fn = U/\sqrt{gL}\) and \(L\) is the length of the vessel.
The sectional hydrodynamic damping is expressed as

\[ D = 2 \sin\left(\frac{\omega^2 B}{2g}\right) \exp\left(-\frac{\omega^2 T}{g}\right) = 2 \sin\left(\frac{1}{2} k B \alpha^2\right) \exp\left(-k T \alpha^2\right) \quad (3.9) \]

The forcing functions \( F, G \) are given by

\[ F = \kappa f \frac{2}{k_e L} \sin\left(\frac{k_e L}{2}\right) \quad (3.10) \]

\[ G = \kappa \frac{24}{(k_e L)^2 L} \left[ \sin\left(\frac{k_e L}{2}\right) - \frac{k_e L}{2} \cos\left(\frac{k_e L}{2}\right) \right] \quad (3.11) \]

where

\[ k_e = |k \cos \beta| \quad (3.12) \]

is the effective wave number and

\[ f = \sqrt{(1 - k T)^2 + \left(\frac{D^2}{k B \alpha^3}\right)^2} \quad (3.13) \]

The Smith correction factor \( \kappa \) is approximated by

\[ \kappa = \exp(-k T) \quad (3.14) \]

Solution Eq.(3.6) and Eq.(3.7) yield the frequency response functions

\[ \Phi_\omega = \eta F \quad (3.15) \]

\[ \Phi_\theta = \eta G \quad (3.16) \]

where

\[ \eta = \sqrt{(1 - 2 k T \alpha^2)^2 + \left(\frac{D^2}{k B \alpha^3}\right)^2} - 1 \quad (3.17) \]

And the phase information of heave can be expressed as

\[ \cos \epsilon_\omega = (1 - 2 k T \alpha^2) \eta \quad (3.18) \]

\[ \sin \epsilon_\omega = -\frac{D^2}{k B \alpha^2} \eta \quad (3.19) \]

The phase information of pitch is shown

\[ \cos \epsilon_\theta = \sin \epsilon_\omega \quad (3.20) \]

\[ \sin \epsilon_\theta = -\cos \epsilon_\omega \quad (3.21) \]

Finally the frequency response function of heave and pitch are represented in Eq.(3.22) and Eq.(3.23). Where the transfer function is set as a complex number \( H \).

\[ \text{Re}\{H_\omega\} = \Phi_\omega \cos \epsilon_\omega; \quad \text{Im}\{H_\omega\} = \Phi_\omega \sin \epsilon_\omega \quad (3.22) \]

\[ \text{Re}\{H_\theta\} = \Phi_\theta \cos \epsilon_\theta; \quad \text{Im}\{H_\theta\} = \Phi_\theta \sin \epsilon_\theta \quad (3.23) \]
3.4 Simulation of ship motions

Since there is no forward speed as assumed before, the responses of the vessel can be defined as a function \( z_m(t) \), which considers heave and pitch motions both. Substitute the transfer function \( H(\omega) \) into the irregular wave expression Eq.(2.14).

\[
\begin{align*}
    z_m(t) &= \sum_{n=1}^{N} (\text{Re}\{H_m(\omega_n)\})(x_{1,n} + i x_{2,n}) (\cos\omega_n t + i \sin\omega_n t) \\
    &= \sum_{n=1}^{N} (\text{Re}\{H_m(\omega_n)\})(x_{1,n} \cos\omega_n t - x_{2,n} \sin\omega_n t) \\
    &\quad - \text{Im}\{H_m(\omega_n)\}((x_{1,n} \cos\omega_n t + x_{2,n} \sin\omega_n t)) \\
    &= \sum_{n=1}^{N} (\text{Re}\{H_m(\omega_n)\}) \cos\omega_n t - \text{Im}\{H_m(\omega_n)\} \sin\omega_n t x_{1,n} \\
    &\quad - (\text{Re}\{H_m(\omega_n)\}) \sin\omega_n t - \text{Im}\{H_m(\omega_n)\} \cos\omega_n t x_{2,n}
\end{align*}
\]

where the transfer function is a complex number \( H(\omega) = \text{Re}\{H_m(\omega_n)\} + i \text{Im}\{H_m(\omega_n)\} \) and variables \( x_1 \) and \( x_2 \) are the wave parameters. \( z_1 \) represents the heave motion and \( z_2 \) represents the pitch motion.

The simulation part is basically finished so far, with wave parameters calculated from generated wave spectrum and transfer functions calculated by closed-form expression, and the next step is the estimation based on the simulated responses.
CHAPTER 4

Estimation of Wave Spectrum

As shown in Figure 3.1, the ship can be treated as a filter when the input is the waves and output is the motions. Inversely, the waves also can be predicted from the ship motions through the filter, which is the final purpose of the thesis. The inverse transfer method is Kalman filtering.

4.1 Kalman filter

Kalman filter is a good method to achieve the goal of running the whole system onboard in real time, when a fast and recursive algorithm is required for spectrum estimation. To conduct the Kalman filtering, a linear dynamical system needs to be built.

4.1.1 Kalman filter algorithm

Consider a discrete time system without control part, whose state at time $k$ is evolved from the period time $k - 1$ state. According to the state equation

$$X(k) = F(k) \, X(k - 1) + \xi(k) \quad (4.1)$$

where
- $X(k)$ is the state vector in time $k$
- $F(k)$ is the state transition matrix
- $\xi(k)$ is the process noise covariance matrix

And the measurements of the system are performed. The output equation of the system is

$$Z(k) = C(k) \, X(k) + \theta(k) \quad (4.2)$$

where
- $Z(k)$ is the vector of measurements
- $C(k)$ is the transformation matrix, which maps the state vector into the measurement domain

- $\theta(k)$ is the measurement noise covariance matrix

First is doing the one step prediction, it is also called time update.

\[
X'(k+1) = F \times X(k) \tag{4.3}
\]
\[
P'(k+1) = F \times P(k) \times F^T + \Xi \tag{4.4}
\]

where $\Xi$ is the process noise covariance matrix, $P(k)$ is the error covariance matrix and $X'$, $P'$ are the predictions of state estimate and estimate covariance.

Last step is the measurement update,

\[
X(k) = X'(k) + K(k)(z_m(k) - C(k)X'(k)) \tag{4.5}
\]
\[
P(k) = (I - K(k)C(k))P'(k)(I - K(k)C(k))^T + K(k)\Theta K^T(k) \tag{4.6}
\]

where $K$ is the Kalman gain.

\[
K(k) = P'(k)C^T(k)(C(k)P'(K)C^T(k) + \Theta)^{-1} \tag{4.7}
\]

and $z_m(k)$ is the measured output, $\Theta$ is noise covariance matrix of measurement.

These five equations Eq.(4.3-7) are one turn of estimation, when time moves one step $k = k + 1$ a new turn will repeat these three steps again. The system will keep iterating with variable $k$.

The calculation process of Kalman filter shows in Figure 4.1
4.1 Kalman filter

Start

Initial error covariance matrix $P(0)$

Predicted (a priori) $P'(k+1)$

Update $K(k+1)$

Updated $P(k+1)$

$k = k + 1$

Time end

Output $P(k+1)$

Initial state estimate $X(0)$

Predicted (a priori) $X'(k+1)$

Calculate the error of prediction

Updated $X(k+1)$

$k = k + 1$

Time end

Output $X(k+1)$

Figure 4.1: Flow chart of Kalman filter

4.1.2 Application of Kalman filter algorithm to wave estimation

Transferring waves to the ship motions can be considered as a linear time varying (LTV) system according to Eq.(3.24), where the estimation of the ship motions is a linear function with variables $x_1$ and $x_2$ of waves represented in Eq.(2.11).

Set $x_1$ and $x_2$ as the state variables because they are the purpose of the estimation to reconstruct the wave spectrum. The output is the motions $z$.

The Kalman filter is an efficient recursive filter, which estimates the system’s state in discrete time. However, the wave system is based on continuous time. The state space needs to be discretized, the continuous time variable $t$ could be rewritten as[3]
\[ t = k \, T_s \] (4.8)

where \( T_s \) is the sampling time of the sensor, which in this simulation can be treated as a constant.

Consider the wave expression Eq.(2.11) and the motion expression Eq.(3.24). The dynamical system in this case does not include the control input, and the noise of measurement and process should be considered in practise. Assume the system states have constant or very slowly varying values compared to the filter dynamics [13], which means Eq.(4.1) and Eq.(4.2) can be transformed into Eq(4.9) and Eq.(4.10)

State equation

\[
\begin{align*}
 x_{1,n}(k+1) &= x_{1,n}(k) + \xi_1(k) \\
 x_{2,n}(k+1) &= x_{2,n}(k) + \xi_2(k)
\end{align*}
\] (4.9)

Output equation

\[
\begin{align*}
 z_n(k) &= (\text{Re}\{H(\omega_n)\} \cos \omega_n t \sin \omega_n t x_{1,n} \\
 &\quad - (\text{Re}\{H(\omega_n)\} \sin \omega_n t \sin \omega_n t x_{2,n} + \theta(k)
\end{align*}
\] (4.10)

where \( \xi_1 \) and \( \xi_2 \) are the process noise, and \( \theta(k) \) is the measurement noise. They are all set as independent Gaussian white noise in this case.

In vector notation, Eq.(4.4) and Eq.(4.5) can be transformed into

\[
\begin{align*}
 X(k+1) &= F \cdot X(k) + \xi(k) \\
 Z(k) &= C(k) \cdot X(k) + \theta(k)
\end{align*}
\] (4.11, 4.12)

Here \( X = [x_{1,n}, x_{2,n}]_n^T \) is the state matrix, which has \( 2 \times N \) states. And \( \xi = [\xi_1, \xi_2]^T \) is the process noise vector.

\[
F = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

is a \( 2 \times 2 \) identity matrix, whose output matrix \( C \) is

\[
C(k) = \begin{bmatrix}
\text{Re}\{H(\omega_j)\} \cos(\omega_n kT_s) - \text{Im}\{H(\omega_j)\} \sin(\omega_n kT_s), \\
- (\text{Re}\{H(\omega_j)\} \sin(\omega_n kT_s) + \text{Im}\{H(\omega_j)\} \cos(\omega_n kT_s))
\end{bmatrix}_n
\] (4.13)

And the target of the estimation is state variable \( X \), the wave spectrum can be reconstruct according to \( X \). The process noise \( \xi_i \) is zero mean white noise with a certain variance \( \delta_\xi^2 \), hence the process noise vector is associated a covariance matrix

\[
\Xi = \begin{bmatrix}
\delta_\xi^2 & 0 \\
0 & \delta_\xi^2
\end{bmatrix}
\]
And the measurement noise $\theta$ is also zero mean white noise with variance $\delta_\theta^2$ and covariance $\Theta = \delta_\theta^2 I$.\[3\]

Before applying the Kalman filtering, the initial data $X(0)$ and $P(0)$ should be set

$$X(0) = [0, 0]^T$$

$$P(0) = \begin{bmatrix} \delta_{x_1}^2 & 0 \\ 0 & \delta_{x_2}^2 \end{bmatrix}$$

where $\delta_{x_1}^2$ and $\delta_{x_2}^2$ are the variances whose values determine the level of uncertainty in the initial condition $X(0)$. The variables and assumptions are substituted into Eq.(4.3-7) to apply Kalman filter.

## 4.2 Forward speed

If forward speed is not considered, the wave frequencies measured from the vessel are true frequencies. With the forward speed, the wave frequencies measured on the vessel will be different from which measured on the ground. The wave frequencies measured from vessel are called encounter wave frequencies.

The relation between encounter frequency and wave frequency is given as

$$\omega_e = \omega - A\omega^2, \quad A = \frac{U}{g} \cos \beta \tag{4.14}$$

Inversely wave frequency can be expressed as

$$\omega = \frac{1}{2A} \left(1 \pm \sqrt{1 - 4\omega_e A}\right) \tag{4.15}$$

While calculating the wave frequencies from encounter wave, it is one-to-one correspondence in head sea and beam sea situations, but the following sea has more correspondences due to Eq.(4.15). It can be seen in Figure 4.2.

Besides, the relation between wave spectrum and encounter wave spectrum is presented as

$$S(\omega)d\omega = S(\omega_e)d\omega_e \tag{4.16}$$
The correspondence between wave frequency and encounter frequency can be expressed as

\[
\begin{align*}
\omega_1 &= \frac{1 - \sqrt{1 - 4A\omega_e}}{2A} \\
\omega_2 &= \frac{1 + \sqrt{1 - 4A\omega_e}}{2A} \\
\omega_3 &= \frac{1 + \sqrt{1 + 4A\omega_e}}{2A}
\end{align*}
\]

The spectrum also needs to be transferred to wave frequency domain. The relation between encounter wave spectrum and wave spectrum can be given as

\[
S(\bar{\omega}_e)\Delta\omega_e = S(\bar{\omega})\Delta\omega
\]

It can be seen that the wave amplitude from corresponding encounter wave frequency \( \bar{\omega}_e \) is same as the wave frequency, which means

\[
\bar{\zeta}_e = \sqrt{2S(\bar{\omega}_e)\Delta\omega_e} = \sqrt{2S(\bar{\omega})\Delta\omega} = \bar{\zeta}
\]
Finally the wave spectrum has to be transferred from encounter wave frequency domain to wave frequency domain. The basic transformation can be seen in Figure 4.3.

\[ A_{e1} = A_{\omega 1} + A_{\omega 3} + A_{\omega 4} \]  

(4.21)

where the \( A_e \) is the energy of encounter spectrum and \( A_\omega \) is the energy of wave frequency. Substitute Eq.(4.16) into Eq.(4.21)

\[ A_{e1} = S(\omega_{e1})\Delta\omega_{e1} = A_{\omega 1} + A_{\omega 3} + A_{\omega 4} \]

\[ = S(\omega_{\omega 1})\Delta\omega_{\omega 1} + S(\omega_{\omega 3})\Delta\omega_{\omega 3} + S(\omega_{\omega 4})\Delta\omega_{\omega 4} \]  

(4.22)

The method of separating the energy from encounter frequency domain to wave frequency domain is base on the data from previous calculation of JONSWAP spectrum.
Because there is no theoretical method, the numerical method should be considered. The $A_{\omega_1}$, $A_{\omega_2}$ and $A_{\omega_3}$ in Eq. (4.22) can be rewritten as

$$
A_{\omega_1} = \frac{S_f(\omega_1)}{S_f(\omega_1) + S_f(\omega_2) + S_f(\omega_3)} A_{e_1} \\
A_{\omega_2} = \frac{S_f(\omega_2)}{S_f(\omega_1) + S_f(\omega_2) + S_f(\omega_3)} A_{e_1} \\
A_{\omega_3} = \frac{S_f(\omega_3)}{S_f(\omega_1) + S_f(\omega_2) + S_f(\omega_3)} A_{e_1}
$$

(4.23)

where the $A_{\omega_i}$ can be described as the function of the wave spectrum $S_f(\omega_1)$, which is an approximate method.
CHAPTER 5

Results and Discussion

In this chapter, an example of spectrum estimation is provided. Container ship S175 is chosen as the vessel and the waves are generated based on the JONSWAP spectrum. Estimations are carried out with respect to one response and two responses, with and without forward speed.

5.1 Numerical Example

5.1.1 Vessel and wave data

The vessel data is mainly used to construct the transfer function, which will be used to generate the simulated responses from the simulated waves and calculate the estimated waves from the simulated responses.

The RAOs data is used in transformation from the waves to the vessel motions, and the same set of data is also used for transferring from the vessel responses to the wave spectrum. Thus the accuracy of transfer functions will not be significant. Main particulars of S175 can be seen in the table 5.1.

<table>
<thead>
<tr>
<th>Table 5.1: Principal dimensions of the S175 container ship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars ($L$)</td>
</tr>
<tr>
<td>Breadth ($B$)</td>
</tr>
<tr>
<td>Depth ($D$)</td>
</tr>
<tr>
<td>Draught ($T$)</td>
</tr>
<tr>
<td>Displacement</td>
</tr>
<tr>
<td>LCG aft of midship</td>
</tr>
<tr>
<td>Block coefficient ($C_B$)</td>
</tr>
<tr>
<td>Midship section coefficient ($C_M$)</td>
</tr>
</tbody>
</table>

The simulation environment has to be built in the beginning of calculation, generating the ship motions in simulated sea state. To get the sea state, $N = 1000$ random wave frequencies $\omega_{simu}$ are generated in range from 0.1 to 2.5 rad/s. And the JONSWAP spectrum $S_J(\omega_{simu})$ is generated by Eq.(2.5) with $H_s=$4m, $T_z=$10.0s. The JONSWAP spectrum and the Bretschneider spectrum will be mainly considered in this case. The shape comparison of these two spectra are shown in Figure 5.1.
5.1.2 Calculation process

The first part is wave environment simulation, the waves can be calculated by substituting the random wave frequencies into Eq. (5.1).

\[ \zeta(t) = \sum_{n=1}^{1000} \sqrt{2S_{J}(\omega_{\text{simu},n})\Delta\omega} \cos(\omega_{\text{simu},n}t + \epsilon_n) \]  

(5.1)

where the phase data \( \epsilon_n \) are random numbers in range \([0, \pi]\).

According to Eq. (2.10), Eq. (5.1) can be rewritten as a function of wave parameters \( x_{\text{simu},1} \) and \( x_{\text{simu},2} \):

\[ x_{\text{simu},1,n} = \sqrt{2S_{J}(\omega_{\text{simu},n})\Delta\omega} \cos(\epsilon_n) \]  

(5.2)

\[ x_{\text{simu},2,n} = \sqrt{2S_{J}(\omega_{\text{simu},n})\Delta\omega} \cos(\epsilon_n + \pi/2) \]  

(5.3)

With the data of waves, ship motions \( z_{\text{simu}}(t) \) can be calculated by use of RAOs. The data of RAOs \( H_m(\omega) \) is given by closed-form expressions.

\[ z_{\text{simu},m}(t) = \sum_{n=1}^{1000} (\text{Re}\{H_m(\omega_{\text{simu},n})\} \cos \omega_{\text{simu},n}t - \text{Im}\{H_m(\omega_{\text{simu},n})\} \sin \omega_{\text{simu},n}t)x_{\text{simu},1,n} \]

\[ - (\text{Re}\{H_m(\omega_{\text{simu},n})\} \sin \omega_{1,n}t - \text{Im}\{H_m(\omega_{\text{simu},n})\} \cos \omega_{\text{simu},n}t)x_{\text{simu},2,n} \]  

(5.4)
When the ship motions are simulated, Kalman filter is used to estimate the wave spectrum. According to Eq.(4.9) to Eq.(4.13), wave frequencies $\omega_{esti}$, encounter frequencies $\omega_{esti,e}$ and transfer function $H$ need to be set before Kalman filter calculation. The wave spectrum can easily be calculated by Eq.(5.5) without forward speed.

$$\frac{x_{esti,1}^2 + x_{esti,2}^2}{2} = S(\omega)\Delta\omega \tag{5.5}$$

Consider the forward speed $U=10$ knots. Due to the difference of calculation from encounter frequencies to wave frequencies in Eq.(4.15), head sea, beam sea and following sea will be considered as different situations.

5.1.2.1 Head sea

When $90^\circ \leq \beta \leq 180^\circ$, which is head sea, bow sea and beam sea conditions, there will be a one-to-one correspondence between wave frequencies and encounter frequencies due to Eq.(4.15). The $N = 40$ wave frequencies $\omega_{esti}$ are set from 0.1 to 2.5 rad/s at $\Delta\omega = 0.06$ rad/s intervals. And the encounter frequencies $\omega_{esti,e}$ can be calculated by Eq.(4.14).

Wave parameters $x_{esti,1}$ and $x_{esti,2}$ can be given by using Kalman filter to estimate waves, with which the wave spectrum $S(\omega)$ can be calculated directly as Eq.(5.3).

$$\frac{x_{esti,1}^2 + x_{esti,2}^2}{2} = (\zeta^2)/2 = S(\omega_e)\Delta\omega_e = S(\omega)\Delta\omega \tag{5.6}$$

5.1.2.2 Following sea

Consider $0 \leq \beta < 90^\circ$, which corresponds to quartering and following sea conditions. Because in following sea, Eq.(4.15) will be in one-to-three correspondence when $\omega_e < g/4U\cos\beta$.

The encounter wave frequencies are set from 0.1 to 2.5 rad/s at 0.06 rad/s intervals as shown in Figure 4.1. And the corresponding wave frequencies can be calculated as Eq.(5.4).

$$\omega = \left| \frac{g}{2U\cos\mu} \right| \left(1 \pm \sqrt{1 \pm \frac{4\omega_e}{g} U \cos \mu} \right) \tag{5.7}$$

With the data of ship responses $z$ and encounter wave frequencies $\omega_e$, the Kalman filter can be applied for the calculation, whose output are wave parameters $x_1$ and $x_2$. The encounter wave spectrum $S(\omega_e)$ can be constructed as in Eq.(5.3).

According to the Eq.(4.21), the method of separating the energy is based on the previous calculation of JONSWAP spectrum. The ratio of the three weights in Eq.(4.22) is set the same as the weights of the JONSWAP spectrum in Eq.(4.23). After separation, the spectrum can be transferred from encounter frequency domain to wave frequency domain.
5.2 Results

The calculation based on the computer with a 3.33GHz Inter Core i5 processor. The simulation time is 800s, it takes average 15s for 40 estimated wave frequencies and 57s for 100 estimated wave frequencies.

5.2.1 RAOs

The data of transfer function is presented as response amplitude operators (RAOs), which is calculated by closed-form method. The two main responses used in this thesis are heave and pitch.

![Heave transfer function for the S175 container ship travelling in different direction](image1)

![Pitch transfer function for the S175 container ship travelling in different direction](image2)
5.2 Results

5.2.2 Estimation results

By using the Kalman filtering method, the waves and responses are estimated. The comparison of the estimated results and the true values are shown in Figure 5.4 and 5.5.

![Figure 5.4: Estimated waves by the Kalman filter](image)

The comparison of estimated and true waves is hard to be evaluated so the spectrum is introduced to describe the results of estimation. As shown in Figure 5.4, in the first 120s (=1200*0.1) the estimated wave is zero due to the setting of initial value. A few seconds is required to get a stable estimation.
The estimation of vessel responses is perfectly matched with the simulated responses.

5.2.3 Estimated spectrum

To examine the results of estimation, the JONSWAP spectrum is used as the generated spectrum. Figure 5.5-8 show the differences of simulations in one response (heave) and two responses (heave and pitch) respectively, with and without forward speed.
5.2 Results

(a) spectrum in 0 degree following sea
(b) spectrum in 45 degrees
(c) spectrum in 90 degrees beam sea
(d) spectrum in 120 degrees
(e) spectrum in 150 degrees
(f) spectrum in 180 degrees head sea

Figure 5.6: Estimation with one response without forward speed in different directions
Figure 5.7: Estimation without forward speed in different directions
Figure 5.8: Estimation with forward speed in different directions

As seen from the figures, the estimated spectrum fits best with the generated
Results and Discussion

In comparison between Figure 5.6 and 5.7, the estimations with two responses show better results than only one response. The causes will be discussed in discussion section.

After considering forward speed, in comparison between Figure 5.7 and 5.8, the energy is lost in the peak of the estimated spectrum in following sea and quartering sea as in Figure 5.7(a) and 5.8(a). There is also energy loss in right side as in Figure 5.7(f) and 5.8(f). However, it shows perfect fit in beam sea.

The coefficient of variation \( c_v \) is introduced to express the error in estimation spectrum.

\[
c_v = \frac{\sigma}{\mu} = \frac{1}{m_0} \int_0^\infty |S(\omega) - S_J(\omega)| d\omega
\]

(5.8)

where \( \sigma \) is standard deviation and \( \mu \) is mean.

Table 5.2: Analysis of data in estimation with one response

<table>
<thead>
<tr>
<th>Direction ( \beta )</th>
<th>0°</th>
<th>45°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height ( H_s ) (m)</td>
<td>3.1</td>
<td>3.4</td>
<td>3.8</td>
<td>3.5</td>
<td>3.2</td>
<td>3.5</td>
</tr>
<tr>
<td>Coefficient of variation ( c_v )</td>
<td>0.23</td>
<td>0.21</td>
<td>0.09</td>
<td>0.13</td>
<td>0.19</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 5.3: Analysis of data in estimation with two responses without forward speed

<table>
<thead>
<tr>
<th>Direction ( \beta )</th>
<th>0°</th>
<th>45°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height ( H_s ) (m)</td>
<td>4.1</td>
<td>3.8</td>
<td>3.8</td>
<td>4.1</td>
<td>4.1</td>
<td>4.2</td>
</tr>
<tr>
<td>Coefficient of variation ( c_v )</td>
<td>0.03</td>
<td>0.11</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 5.4: Analysis of data in estimation with two responses with forward speed

<table>
<thead>
<tr>
<th>Direction ( \beta )</th>
<th>0°</th>
<th>45°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height ( H_s ) (m)</td>
<td>3.3</td>
<td>3.7</td>
<td>3.8</td>
<td>3.2</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>Coefficient of variation ( c_v )</td>
<td>0.23</td>
<td>0.17</td>
<td>0.09</td>
<td>0.34</td>
<td>0.46</td>
<td>0.49</td>
</tr>
</tbody>
</table>

where the significant wave height \( H_z = 4m \).
5.3 Discussion

5.3.1 Range of estimated spectrum

The estimation result varies due to the setting of parameters. The range of estimated spectrum can be drew by running 50 times of simulation in following sea with forward speed \( U = 10 \) knots.

![Graph comparing spectrum with two responses of heading 30 degrees](image)

Figure 5.9: Range of estimation results (The figure will be redraw later)

The black line and green line show the maximum and minimum boundary of the estimated spectrum respectively.

5.3.2 Transformation between encounter frequency domain and wave frequency domain

The method to transfer the spectrum from encounter frequency domain and wave frequency domain is based on the numerical distribution in the JONSWAP spectrum in Eq.(4.23). When the generated spectrum is the JONSWAP spectrum, the estimated spectrum will surely show good performance. In this section, the Bretschneider spectrum is used as generated spectrum to examine the transfer method. The Figure 5.10-5.12 show the spectrum in quartering and following sea.
Figure 5.10: spectrum in 0 degrees following sea

Figure 5.11: spectrum in 30 degrees

Figure 5.12: spectrum in 60 degrees
5.3 Discussion

As seen in the figures, the estimated spectra show the acceptable results. The transformation method can be used not only in JONSWAP spectrum but also other relevant spectra.

5.3.3 Energy lost in the estimated spectrum

Compare Figure 5.6(a) and 5.7(a). The energy loss is mostly on the wave frequency range of \([0.6, 1]\) rad/s, where it can be find the same situation in comparison between Figure 5.7(f) and 5.8(f).

It can be seen that this loss is in the one response and in head sea with forward speed. The main reason is based on RAOs data. It shows in Figure 5.13.

In Figure 5.13(a), the mostly energy of heave response is in the area where wave frequency is lower than 0.6 rad/s. This is why the energy is lost in one response case.

And in Figure 5.13(b), the mostly energy of pitch response is in the area where wave frequency is lower than 0.7 rad/s when in head sea. The area moves left with direction \(\beta\) rises and with forward speed. This is why the energy lost in two responses with forward speed.

Compare Figure 5.7(a) and 5.8(a). Energy is lost in estimated spectrum peak. The transformation from wave frequency domain to encounter frequency domain should be introduced.

Figure 5.17 and 5.18 show the transformation of energy in different regions wave frequency domain to encounter frequency domain.

As Figure 5.7 shows, the loss of energy occurs in the region II, where the main reason of the energy loss lies in the missing of wave frequency points in region II.
Figure 5.14: Wave spectrum showing regions for encounter spectrum calculation[6]

Figure 5.15: Wave encounter spectrum corresponding to Figure 5.21 in following sea[6]

It can be seen in Figure 5.15, region II has the most energy in the spectrum but has
the same number of frequencies in Figure 5.14. With the constant interval between these estimated wave frequencies, energy will lose during the transformation.
6.1 Conclusion

The main purpose of this thesis is to investigate the Kalman filtering applied to wave spectrum estimation. The method is used for real-time estimation of onboard ships, so the test of the method should be conducted in simulation environment. The simulated sea state is generated by the JONSWAP spectrum and the simulated vessel responses are calculated by the simulated sea state and transfer functions.

Thus, the transfer function are calculated by closed-form method, and the same RAOs data are used in the Kalman filter.

The spectrum estimation method is evaluated using simulated data for container ship S175 as a case study. The procedure is examined by the JONSWAP spectrum. The significant wave heights and the peak periods are calculated to compare with the true values, where they are really close, which shows that the estimation results are acceptable.

Regarding to the numerical method of transferring the encounter spectrum to the wave spectrum in following sea condition, the Bretschneider spectrum is used to examine the accuracy of the method. The results are reasonable according to Figure 5.10-12. Two types of energy loss problem is analysed, one is due to RAOs data, one is because the missing of wave frequencies.

In general, this study proves the practice of the Kalman filtering used in estimation of spectrum based on the simulated responses. There might lack accuracy in estimation procedure because of possible errors in acquiring data during evaluating spectrum.

6.2 Future work

6.2.1 Full-scale data

The estimation of this thesis is based on the simulation data and it will be practical to be tested by full-scale data.
6.2.2 Strip theory

In this thesis, the data of RAOs is calculated by closed-form method, which is not accurate. It will not be a problem when the estimation is based on the simulated responses. However, the accuracy of RAOs is required if the estimation is conducted on measured responses.

Strip theory is a popular approximation of the 3-D Neumann-Kelvin formulation for vessel, and the vessel is expected to have significant forward speed.

In strip theory, the vessel has to be considered as long and slender body. Figure 6.1 shows how to make the strip theoretically.

![Figure 6.1: Strip theory](image)

6.2.3 Short crested waves

In this case, the estimation focus on long crested wave, which is only one direction. In real sea state, the waves are mixed by different direction, where the wave spectrum should be included in the direction information. And a simple example is shown in Figure 6.2.

![Figure 6.2: Estimation of directional spectrum](image)
where the transfer function calculated at 30° intervals. The spreading parameter is set as 2. The basic wave parameters \((H_s, T_z, \mu)\) and wave frequencies are the same as the formal example.
[15] SMC IMU. SMC Ship Motion Control Ltd. 2015.

